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ABSTRACT

In June 1984, the State Board of Education adopted the Oregon Action Plan for Excellence which established the direction for school improvement in the state for the next decade. A central concept of the Action Plan is that while the state will determine what must be taught in public schools, the schools will determine how it will be taught. This document is intended to provide the essential information which local districts need to merge state curriculum expectations with their own local determinations for mathematics K-12. The report contains the following sections: (1) introduction; (2) executive summary of content strands; (3) grade level teaching emphasis; (4) instructional themes; (5) staff development; (6) organizational patterns and teaching considerations; (7) comprehensive goals and outcomes (comprising more than half the document and charting expectancies for grades K through 8 as well as grade 11); and (8) bibliography. (RH)

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MATHEMATICS

Comprehensive Curriculum Goals

A Model for Local Curriculum Development

October 1987

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OREGON SCHOOLS--A TRADITION OF EXCELLENCE

SE 048 679

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FOREWORD

In June 1984 the State Board of Education adopted the Oregon Action Plan for Excellence which established the direction for school improvement in the state over the next decade. The Action Plan drew upon the insights of teachers, administrators, school board members and community and business leaders.

A central concept of the Action Plan is that while the state will determine WHAT must be taught in public schools, the schools will determine HOW it will be taught. This document is intended to provide the essential information which local districts need to merge state curriculum expectations with their own local determinations for Mathematics.

All who have joined in the spirit of the Action Plan for Excellence have shared a commitment to high-quality performance. We are continuing to learn about how to provide children with the very best in public education, and we welcome your comments and questions. For further information about this guide, contact the Specialist for Mathematics Education, 378-3566.

Verne A. Duncan
State Superintendent
of Public Instruction

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Two major curriculum documents served as the primary basis for the development of this publication: (1) Essential Learning Skills and (2) The Oregon Mathematics Concept Papers. The concept papers were developed as an Oregon Mathematics Education Council (OMEC) curriculum project. Nearly 100 volunteers worked on their own time for about two years in study, debate, presentations, and writing. Their conclusions present "forward-looking" ideas for school mathematics. Their names and schools at the time the OMEC curriculum project was launched are listed below.

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INTRODUCTION

THE OREGON ACTION PLAN FOR EXCELLENCE

The Action Plan identified seven areas of improvement, one of which called for a statewide definition of what students should learn:

> The Oregon Department of Education, working with local school districts and higher education institutions, shall define the required common curriculum goals for elementary and secondary schools in terms of the learning skills and knowledge students are expected to possess as a result of their schooling experience.
>
> Local school districts, with assistance from the Oregon Department of Education, shall be responsible for organizing the curriculum and delivering instruction to achieve the common curriculum goals.

Common Curriculum Goals

The first stage in defining the Common Curriculum Goals was to develop the Essential Learning Skills--the basic skill and performance expectations for all students in the areas of reading, writing, speaking, listening, mathematics, reasoning and study skills. The second stage was to develop Common Knowledge and Skills in individual subject areas. Together with the Essential Learning Skills, they form the Common Curriculum Goals for all students.

A. Essential Learning Skills

The Essential Learning Skills are considered basic to all students' learning, and all teachers are expected to provide instruction in these skills. Only to the degree that students develop these skills and form the habit of using them, can instruction in subject matter areas be successful. The skills are not specific to any one discipline but provide a link across all disciplines. Furthermore, the skills do not grow in isolation from content; they are strengthened through practice and use in all subject areas.

B. Common Knowledge and Skills

Looking beyond the Essential Learning Skills, MATHEMATICS: Common Curriculum Goals defines more fully the essentials in a strong mathematics program for all Oregon public schools.

C. Comprehensive Mathematics Curriculum

Each district will want to extend and elaborate upon the skills outlined in Essential Learning Skills and MATHEMATICS: Common Curriculum Goals in order to create its own unique, comprehensive mathematics curriculum. Students should have the opportunity to demonstrate their achievement in a variety of ways. Equal opportunity to learn and the special needs of students are primary considerations in determining acceptable performance levels. Mathematics: Comprehensive Model Curriculum offers specific suggestions for district consideration in meeting such student needs.

In anticipating the three-stage curriculum development by the Department, and in response to the national awakening of the growing need to update school mathematics, the Oregon mathematics education community began work in the spring of 1984. After more than two years of development work facilitated by the Oregon Mathematics Education Council The Oregon Mathematics Concept Papers were published in the summer of 1986. The concept papers summarize the national literature and translate it into recommendations for an updated mathematics curriculum. Both Common Curriculum Goals and Comprehensive Model Curriculum include many ideas from the concept papers.

State Standards

The Common Curriculum Goals receive their authority from the Oregon State Standards for Public Schools, OAR 581-22-420 and 581-22-425. These rules were amended by the State Board of Education in January 1986. The Essential Learning Skills deemed appropriate for mathematics instructional programs are included in the Common Curriculum Goals. This document on Comprehensive Curriculum Goals builds upon the first two by offering suggestions to districts for developing their own comprehensive program which meets Standards 581-22-402, 420 and 425 in the program area of mathematics.

PHILOSOPHY/RATIONALE UNDERLYING THIS CURRICULUM

The 1980s have been a time of educational reform in the nation at large. There has been a strong and pervasive quest for excellence and equity in education in general, and mathematics education in specific. The Oregon Action Plan for Excellence established the direction for school improvement in the state and the Essential Learning Skills document outlined the common skills across all program areas for elementary and secondary education. MATHEMATICS: Common Curriculum Goals, was written in relationship to the preceding documents specifically, as well as to the reform effort generally. MATHEMATICS: Comprehensive Model Curriculum, the third stage of curriculum development driven by the Oregon Action Plan for Excellence, builds on MATHEMATICS: Common Curriculum Goals and offers many specific suggestions for districts to consider in developing their comprehensive programs. Together the three documents present a curriculum plan for an updated mathematics program to meet the needs of a technology-based society. Thoughtful readers will note that these three curriculum publications describe major changes in school mathematics.

Although the underlying principles of mathematics are constant, the optimum structure for the presentation and use of mathematics has been shifting in response to the rapidly expanding importance of technology in solving problems. Today's world demands the ability to think and use mathematical ideas to solve problems and to make decisions. The time our pupils spend learning mathematics can no longer be limited to practicing long, repetitive or tedious procedures which are more efficiently done with calculators. The impact of technology and its implications for mathematics education are reflected in this document.

The increasingly common uses of calculators underlines the need for mental computations and estimations. Although the development of these skills has always been implicit in mathematics instruction, they have not always been taught systematically and fully. Deliberate and

thorough development of the ability to estimate and do mental arithmetic is a regular part of the computational strand at all school levels in the curriculum outlined in this document.

Even though the calculator is becoming an increasingly available tool for problem solving, its use is of little value if the user has inadequate conceptual understanding of number and operation. To help students understand "which buttons to push," the curriculum must place significant emphasis on the concepts which underlie basic mathematical skills. Increasingly, research supports the use of concrete materials as models for number concepts and for operations. The curriculum outlined in this document provides that a broad range of manipulatives be used to introduce new concepts at all levels. In addition to going from concrete to the abstract, opportunities are provided for finding concrete representations for abstract concepts and their symbolic representations. The use of manipulatives as tools for increasing understanding extends naturally to their use as tools to assist in learning problem-solving skills. The use of concrete materials as problem-solving tools is incorporated into a variety of curriculum content areas as outlined in this document.

As hardware becomes both more sophisticated and less expensive and as software development continues to expand and become more "user friendly," computers will inevitably become major tools for mathematics education. The value of computers in creating geometric displays, organizing and graphing data, simulating real-life situations, and generating numerical sequences is recognized in this document.

Mathematics is about making sense of the world. The mathematics outlined in this document is consistent with the nature of the subject. This means that pupils are learning mathematics in ways that allow them to explore relationships and to develop understandings. The fundamental premise on which this document is based is that every aspect of school mathematics that pupils encounter should enhance their understanding of mathematical ideas and promote the growth of thinking.

Organization

In order to provide a curriculum consistent with the philosophy outlined above, the common curriculum goals for math have been organized into nine content strands. They are:

1.0 Number and Numeration: Students demonstrate an understanding of number and numeration concepts and use these understandings to interpret and solve problems.

2.0 Appropriate Computational Skills: Students select and use the most appropriate form of computation - manipulative, mental, paper/pencil, estimation or calculator usage to solve problems and check all computations for reasonability.

3.0 Problem Solving: Students use problem-solving skills and strategies to solve routine and nonroutine problems.

4.0 Geometry and Visualization Skills: Students recognize geometric patterns and relationships and apply them in solving problems and making predictions.

5.0 Measurement: Students measure quantities and use measurements to keep records, solve problems and make predictions.

6.0 Statistics and Probability: Students collect, organize, record and interpret data, and be able to predict probable outcomes based on collected data.

7.0 Mathematical Relationships: Students recognize and use number patterns, relationships and logical thinking skills to make predictions and to solve problems.

8.0 Oral and Written Communication Skills: Students use vocabulary, speech, numerals and other symbol systems essential for effective individual and group problem solving, and for effective oral and written communication of mathematical concepts, problem-solving processes and results.

9.0 Appropriate Study Skills: Students select and use appropriate study skills in order to accomplish mathematical learning tasks.

The content outlined in each content strand includes both the essential learning skills deemed appropriate for mathematics instruction and the common curriculum outcomes unique to mathematics.

EXECUTIVE SUMMARY OF CONTENT STRANDS

The content of mathematics programs is organized under nine content strands. This section presents a brief summary and rationale for each strand.

1. Number and Numeration: Basic Concepts, Principles and Meanings

   The mastery of a particular bit or area of knowledge at a level that makes it genuinely functional in one's life requires that it be understood thoroughly, that it be connected to related bodies of thought, and that it be integrated with other knowledge/attitudes/perceptions. Applied to mathematics specifically, this means that students need to apply in a conscious way the basics of number and numeration if they are to be masterful users of the algorithms they are learning. To assist in this objective the mathematics content provides opportunities to build concepts and demonstrate understandings through the use of concrete models for whole number, fraction and decimal numerations.

2. Appropriate Computational Skills

   Mastery of appropriate computational skills is a necessary outcome of a mathematics program. Computational skills need to be related to real-world situations and seen as a means of enhancing a person's ability to use mathematics in daily living. Memorization of all one-digit basic facts at the quick recall level is imperative. Appropriate computational skills include mental arithmetic, estimation, and calculator use as well as paper/pencil computation. Instruction should include opportunities to select the more appropriate mode of computation and to determine whether or not answers are reasonable. Furthermore, the instructional content provides opportunities to demonstrate conceptual understandings and reasonability of answers through the use of concrete models and materials.

3. Problem Solving

   Problem solving has been designated as the central goal for mathematics. A problem is a perplexing situation in which an individual or group accepts the challenge of finding ways to clarify or resolve the difficulties involved. Frequently, the problem can be approached in many ways. Occasionally, some of the resulting investigations are nonproductive. Sometimes they are so productive as to lead to many different solutions or suggest more problems to solve.

   Problem-solving skills and strategies should be explicitly emphasized and problem solving should be incorporated frequently into the approaches used in teaching the required topics throughout the grade levels.

4. Geometry and Visualization Skills

   We live on a sphere called Earth and work in a three-dimensional world. Citizens, consumers, and workers require some knowledge of geometry. Much more knowledge is needed in certain occupations such as plumbing,

carpentry, forestry, interior decorating, architecture and engineering. In addition then to acquiring knowledge of certain geometric concepts and properties and their applications, students need to develop their spatial and visualization skills.

The instructional content emphasizes exploration activities, informal reasoning, and the use of problem-solving skills. Much use is made of tools which aid in geometric explorations, including the compass, protractor, the straightedge, squared (grid) paper, and the computer.

5. Measurement

Measurement concepts surround us. We use an understanding of measures to quantify and interpret the world. Modern technology is totally dependent upon measurement. Hidden in most of humanity's spectacular accomplishments are innumerable measurements, each related to or dependent upon a myriad of other measurements.

Measurement can be taught more successfully if estimation is a content objective. To teach measurement we should be concerned with more than a system of measures. We must teach a "doing" kind of mathematics. Activity gives meaning to measuring skills, makes the resultant learning personally satisfying to the pupil and begins the development of a process that will be used throughout life.

6. Statistics and Probability

We live in a society in which we are confronted daily with quantitative information or data. Statistics is the science or study of data. Quantitative literacy is a requirement for all educated individuals who want to be informed citizens and hold jobs in technical businesses and industries. Throughout life, decisions about health, citizenship, parenthood, employment, financial concerns, and sports are based upon quantitative information. Statistics is most often concerned with using information in the face of uncertainty. Probability gives us a way to measure uncertainty.

In addition, studying statistics and probability can help in the development of students' critical thinking skills. In carrying out experiments, students develop ways to cope with uncertainty as they are searching for truth in a situation and learning to report it faithfully. Approaching situations statistically can help make students face up to their own prejudices, think more consistently about arguments, and justify their thinking with numerical information.

7. Mathematical Relationships

Topics in this content strand are used to show how one thing changes as another changes. Formulas, tables, and graphs often have close relationships to the "real world" and frequently can be used in making useful predictions. Ratio, proportion, and percent are also used extensively in problem-solving situations. Program content emphasizes patterning, using logical thinking skills, modeling of concepts and real world applications.

8. Oral and Written Communication Skills

Two emerging trends are reflected in "Common Curriculum Goals" and "Comprehensive Model Curriculum" for math; integration of learning across content areas, and the development and use of enabling skills which help students learn mathematics and other disciplines. The strand, "Oral and Written Communication Skills" offers the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

It seems fitting that the ninth content strand be "Appropriate Study Skills." A full repertoire of study skills enables students to learn school mathematics efficiently while becoming independent learners. Only by reaching this level can students become life-long consumers and users of mathematics.

GRADE LEVEL TEACHING EMPHASES

Kindergarten

1. Number and Numeration

   Initial number and numeration activities include numerous counting experiences using actual objects. Pupils demonstrate comprehension of whole numbers up to 10 by reading, writing, ordering, modeling and comparing. They also use concrete models to show simple place value and to demonstrate halves and wholes. Other number experiences include skip counting by 5 and 10; using ordinals (1st to 5th); and estimating the number of actual objects and checking reasonableness of answers by counting.

2. Appropriate Computational Skills

   Computational activities are broad-based and include many experiences using manipulatives, estimation and mental arithmetic. The emphasis is on helping pupils relate real experiences with the abstract symbols that represent those activities, rather than on automatic answers to written exercises.

3. Problem Solving

   Pupils are provided with opportunities to use concrete materials to solve problems. They work cooperatively toward the solution of problems and discuss and compare strategies and solutions. Role playing with "real world" objects is used to solve simple oral problems and to model "put together" and "take away" situations. Guessing and checking, making a model, generating a pattern and drawing a picture are other problem-solving skills to which pupils are introduced.

4. Geometry and Visualization Skills

   Emphasis is placed on exploration activities with both plane and solid shapes. Pupils use a variety of manipulative materials, recognize geometric patterns and applications in the environment, and sharpen visualization skills.

5. Measurement

   Emphasis is placed on "hands on" activities which help develop measurement concepts and understandings of pupils have numerous informal experiences estimating and determining length, weight and capacity using nonstandard uniform units. They explore money, time and temperature concepts as they occur in daily situations.

6. Statistics and Probability

   Pupils have numerous informal experiences with activities that have outcomes that depend on chance. They collect and record data using tally marks, lists and simple charts; and make and draw conclusions from real graphs (an array of actual objects on a rectangular grid), picture graphs and charts.

7. Mathematical Relationships

   Pupils sort and classify actual objects and reproduce a pattern in a sequence of actual objects. They identify characteristics of

simple objects that remain the same even though some change occurs; state relationships using terms such as "greater than" and "less than"; and describe in simple terms how a solution was reached.

8. Oral and Written Communication

   Pupils use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

   Pupils use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade One

1. Number and Numeration

   Pupils explore number and numeration concepts through counting and one-to-one correspondence activities with actual objects. They demonstrate comprehension of whole numbers to 20 by reading, writing, ordering, modeling, and comparing. Concrete models are used to demonstrate an understanding of place value grouping and exchanges and to show halves and wholes. Other number and numeration activities include rote counting to 20; skip counting by 2, 5 and 10; counting on and counting backward; estimating the number of objects and checking the reasonableness of answers by counting; using ordinals (to 10th) and demonstrating addition and subtraction properties.

2. Appropriate Computational Skills

   Pupils have experiences learning to calculate using manipulatives, estimation, mental arithmetic, calculators and paper/pencil methods. Considerable attention is given to manipulating concrete objects and to connecting "real world" experiences to the abstract mathematical symbols which represent them.

3. Problem Solving

   Problem-solving skills such as guessing and checking, using concrete objects, making a model, generating a pattern and drawing a picture are emphasized. Pupils have many opportunities to engage in cooperative problem solving and to discuss and compare strategies and solutions. Role playing with concrete objects is used to solve simple word problems and pupils create their own story problems to match addition and subtraction number sentences.

4. Geometry and Visualization Skills

   Emphasis is placed on "hands-on" activities with both plane and solid shapes. Pupils use a variety of manipulative materials, copy and extend geometric patterns, recognize geometric applications in the environment and sharpen visualization and estimation skills.

5. Measurement

   Emphasis is placed on the act of measuring and on "hands-on" activities which help

develop measurement concepts and understandings. Pupils have numerous informal experiences estimating and determining length, weight and capacity using nonstandard units. They explore time and temperature concepts as they occur in daily situations and identify and order coins (penny, nickel and dime) by value.

6. Statistics and Probability

   Pupils have numerous informal experiences with activities that have outcomes that depend on chance and make predictions about simple future outcomes. They collect and record data using tally marks, lists and simple charts; and make and draw conclusions from real graphs, picture graphs, and charts.

7. Mathematical Relationships

   Pupils sort and classify actual objects and reproduce and extend patterns in a sequence of objects or numbers. They state relationships using such terms as "greater than," and "less than", and describe in simple terms how a solution was reached.

8. Oral and Written Communication

   Pupils use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

   Pupils use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade Two

1. Number and Numeration

   Pupils demonstrate comprehension of whole numbers to 100 by reading, writing, ordering, modeling, and comparing. They use concrete models to demonstrate two-digit place value understandings, grouping and exchanging. Place value experiences also include activities in which pupils estimate the number of actual objects and check answers by grouping and counting. Other number and numeration activities include using concrete models to demonstrate halves, wholes, thirds, and fourths; rote counting to 100; skip counting by 2, 5 and 10; and demonstrating addition and subtraction properties.

2. Appropriate Computational Skills

   Pupils have opportunities to use all appropriate forms of computation including manipulatives, estimation, mental arithmetic, calculators, and paper/pencil. Instruction is provided in thinking strategies to assist in basic addition and subtraction facts. Pupils deal with addition and subtraction problems together so they can compare the two processes and make sense of them. Early experiences in two-digit addition/subtraction are modeled using concrete materials and include situations in which regrouping may or may not be required so that pupils learn from the beginning to always check to determine if regrouping is necessary.

3. Problem Solving

Problem-solving skills such as guessing and checking, using concrete objects, making a model, generating a pattern and drawing a picture are emphasized. Pupils have many opportunities to engage in cooperative problem solving and to discuss and compare strategies and solutions. Role playing with concrete objects is used to solve simple word problems and pupils create their own story problems to match addition and subtraction number sentences and algorithms. Opportunities to apply problem-solving skills also occur in drill and practice activities such as games and puzzles.

4. Geometry and Visualization Skills

Emphasis is placed on "hands-on" activities with both plane and solid shapes. Pupils use a variety of manipulative materials to develop an understanding of perimeter, area and volume concepts. They copy and extend patterns, recognize geometric applications in the environment, and sharpen visualization and estimation skills.

5. Measurement

Emphasis is placed on the act of measuring and on "hands-on" activities which help develop measurement concepts and understandings. Pupils estimate and measure length, weight and capacity using non-standard units and begin to use standard metric and customary units to measure length. They identify, order and use the U.S. coins and recognize and use time and temperature concepts.

6. Statistics and Probability

Pupils have numerous experiences with activities that have outcomes that depend on chance; make predictions using such terms as "best chance," "fair," "not likely"; apply intuitive probability concepts in games and activities and predict simple future outcomes. They collect and record data and make and draw conclusions from real charts; picture graphs, bar graphs, and charts.

7. Mathematical Relationships

Pupils sort and classify actual objects and recognize, reproduce and extend object patterns and number patterns in charts and tables. They state relationships using such terms as "greater than," "less than," and "equal to."

8. Oral and Written Communication

Pupils use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

Pupils use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade 3

1. Number and Numeration

   Students read, write, order, and compare whole numbers to 1000, commonly used fractions, and decimals in tenths and hundredths (using money models). They use concrete materials to model, place value exchanges, whole number place values to 1000; commonly used fractions; and decimals. Place value concepts are used to estimate and sense reasonableness of answers. Properties of addition, subtraction, and single-digit multiplication are modeled with actual objects, used and applied.

2. Appropriate Computational Skills

   Students have opportunities to select and use all appropriate computational skills including manipulatives, estimation, mental arithmetic, calculators, and paper/pencil. Mastery of basic addition and subtraction facts is encouraged and pupils are provided with thinking strategies to assist with meaningful acquisition of multiplication facts. Concrete materials continue to be used to model addition and subtraction algorithms and to demonstrate the various meanings of multiplication. Instruction is provided in specific appropriate mental arithmetic skills (such as adding and subtracting multiples of tens or hundreds) and pupils are encouraged to use rounding and other skills to make approximate calculations and to check all answers for reasonability.

3. Problem Solving

   Problem-solving skills such as guess and check, use concrete objects, make a model, look for a pattern, and draw a picture are emphasized. Students have many opportunities to engage in cooperative problem solving and to discuss and compare strategies and solutions. They solve onestep word problems, including those involving money and measurements, and create their own word problems to match addition, subtraction, and multiplication algorithms. Opportunities to apply problem-solving skills also occur in drill and practice activities such as games, puzzles, calculator activities, and challenge problems.

4. Geometry and Visualization Skills

   Emphasis is placed on "hands-on" activities with both plane and solid shapes. Students use a variety of manipulative materials to develop an understanding of perimeter, area, and volume. They copy and extend geometric patterns, make simple constructions, recognize geometric applications in the environment, and sharpen visualization and estimation skills.

5. Measurement

   Emphasis is placed on the act of measuring and on activities which help develop measurement concepts and understandings. Students estimate and then determine length and weight (mass) using nonstandard, metric, and customary units of measure and select

appropriate instruments and units for specific measurement tasks. They identify, order, and use all U.S. coins; reading time using standard and digital clocks; and estimate, read and record temperature in C° and F°.

6. Statistics and Probability

   Students have numerous experiences with activities which have outcomes that depend on chance; make predictions about simple possible future outcomes; and apply intuitive probability concepts in games and activities. They collect and record data and make and draw conclusions from bar graphs and picture graphs.

7. Mathematical Relationships

   Students sort and classify actual objects and identify number patterns in charts and tables. They use number patterns and relationships to make predictions, make simple tables of values and state mathematical relationships using terms such as "less than" and "equal to."

8. Oral and Written Communication

   Students use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

   Students use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade Four

1. Number and Numeration

   Students demonstrate an understanding of place values to one million and commonly used fractions and decimals to hundredths by modeling with concrete and pictorial materials and by reading, writing, ordering and comparing. They demonstrate, use and apply properties of addition, subtraction, multiplication and single-digit division with whole numbers; recognize and use concepts related to multiples and factors; and recognize and use appropriate mathematical terms such as product, sum, remainder, and quotient.

2. Appropriate Computational Skills

   Students have opportunities to select and use all appropriate computational skills including manipulatives, estimation, mental arithmetic, calculators, computers, and paper/pencil. Immediate recall of all basic facts is essential and is assisted by a thorough review and application of appropriate thinking strategies. Concrete materials are used to model multiplication and division of whole numbers, addition and subtraction of decimals, and addition and subtraction of fractions with like denominators. Considerable attention is given to the development and use of mental math skills, including operations with powers and multiples of 10. Pupils are encouraged to use rounding and other skills to make approximate calculations and to check all answers for reasonability.

3. Problem Solving

Problem-solving skills such as guess and check; look for a pattern, make a systematic list; and make a model or drawing are emphasized. Students have many opportunities to engage in cooperative problem solving and to discuss and compare problem-solving strategies and solutions to routine and nonroutine problems. They solve one-step word problems, including those involving money and measurements, and create their own story problems to match addition, subtraction, multiplication and division algorithms. Opportunities to apply problem-solving skills also occur in drill and practice activities such as games, puzzles, calculator activities and challenge problems.

4. Geometry and Visualization Skills

Emphasis is placed on "hands-on" activities with both plane and solid shapes. Students use a variety of manipulative materials to estimate and then determine perimeter, area and volume. They copy and extend geometric patterns, make simple geometric drawings and constructions, recognize geometric applications in the environment and sharpen visualization skills.

5. Measurement

Emphasis is placed on the act of measuring and on estimation and other activities which help develop measurement concepts and understandings. Students recognize and use meters, centimeters, feet, yards, inches, grams, kilograms, ounces and pounds, and select the most appropriate instrument and unit for a specific measurement task. They identify, order and use U.S. currency; read time using standard and digital clocks; and estimate, read and record temperature in C° and F°.

6. Statistics and Probability

Students participate in probability experiments and game activities which have outcomes that depend on chance, recognize the concept of fair or unfair, and make predictions about simple possible future outcomes. They make and draw conclusions from charts, picture graphs, bar graphs, and computer generated graphs and tables.

7. Mathematical Relationships

Students sort and classify actual objects and simple geometric figures and identify number patterns in charts and tables. They use number patterns and relationships to make predictions, make simple tables of value, state mathematical relationships, and use the symbols >, < and =.

8. Oral and Written Communication

Students use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

Students use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade Five

1. Number and Numeration

Students demonstrate an understanding of place values to one million; commonly used proper fractions, improper fractions, and mixed numbers; and decimals to thousandths by modeling with concrete and/or pictorial materials and by reading, writing, ordering, and comparing. They demonstrate and use properties of whole number operations; recognize and use appropriate mathematical terms; and recognize and use concepts related to multiples, factors, primes, and composites.

2. Appropriate Computational Skills

Students have opportunities to select and use all appropriate computational skills including estimation, mental arithmetic, calculators, computers, and pencil/paper. They apply acquired strategies to aid in the quick recall of all basic facts. Concrete materials are used to model the various meanings of whole number multiplication and division and to interpret remainders. Models such as money or metrics are used to demonstrate addition and subtraction of decimals and multiplication and division of decimals by whole numbers. Concrete models are also used to demonstrate an understanding of the addition and subtraction of commonly used fractions and the multiplication of a fraction and whole number using the "of" concept. Students solve mentally appropriate whole number, decimal and fraction problems and use a calculator and/or computer to solve problems with lengthy calculations or large numbers. They use rounding and other techniques useful in mental computation to make approximate whole number, fraction and decimal computations and to check reasonability of all answers.

3. Problem Solving

Problem-solving skills such as guess and check, look for a pattern, make a systematic list, make a drawing or model, and solve a simpler related problem are emphasized. Students have many opportunities to engage in cooperative problem solving and to discuss and compare problem-solving strategies and solutions to routine and nonroutine problems. They solve one- and two-step word problems and create their own story problems to match whole number, fraction and decimal algorithms. They also have opportunities to apply problem-solving skills in drill and practice activities such as games, puzzles, calculator and computer activities, and challenge problems.

4. Geometry and Visualization Skills

Emphasis is placed on "hands-on" activities with both plane and solid shapes. Students use a variety of manipulatives to estimate and then determine perimeter, area and volume. They identify properties of common geometric figures, make geometric drawings and constructions, recognize geometric applications in the environment and sharpen visualization skills.

5. Measurement

Emphasis is placed on the act of measuring and on estimation and other activities which help develop measurement concepts and understandings. Students recognize and use metric and customary units to estimate and determine length and weight mass. They select the most appropriate unit for a specific measurement task; convert among units of length within the same measurement system; use U.S. currency; read and record temperature; and determine indirect measurements by locating points and giving coordinates of points on maps.

6. Statistics and Probability

Students generate, record and interpret data from probability experiments, predict chances of an outcome, and recognize certain and impossible probabilities. They use charts, tables and lists to organize all possible outcomes of an experiment; and read, interpret, construct and draw conclusions from bar graphs, line graphs, tables, and charts.

7. Mathematical Relationships

Students sort and classify actual objects and simple geometric figures and identify number patterns in charts and tables. They use number patterns and relationships to make predictions, make and interpret tables of value, state mathematical relationships, and use the symbols >, < and =. Decimal and fraction understandings and 100-grids are used to illustrate the "for every hundred" model of percent.

8. Oral and Written Communication

Students use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

Students use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade Six

1. Number and Numeration

Students demonstrate an understanding of place value to one million; commonly used proper fractions, improper fractions, and mixed numbers; and decimals to thousandths by modeling with concrete and/or pictorial materials and by reading, writing, ordering, and comparing. They apply and use properties of whole number operations; recognize and use appropriate mathematical terms, grouping symbols, and order of operations and divisibility rules; and recognize and use concepts related to multiples, factors, primes, and composites.

2. Appropriate Computational Skills

Students have opportunities to select and use all appropriate computational skills including estimation, mental arithmetic, calculators, computers, and pencil/paper. Concrete and pictorial models such as money and grid paper are used to model decimal and fraction operations. Students solve

mentally appropriate whole number, decimal and fraction problems and convert mentally between commonly used fractions and decimals. They use a calculator and/or computer to solve problems with lengthy calculations or large numbers. Instruction in skills useful in making approximate whole number, decimal, and fraction calculations is provided and these skills are applied in checking the reasonability of all computational results.

3. Problem Solving

Problem-solving skills such as guess and check, look for a pattern, make a systematic list, make a drawing or model, and solve a simpler related problem are emphasized. Students engage in cooperative problem solving and discuss and compare strategies and solutions to routine and nonroutine problems. They solve one- and two-step word problems and create their own story problems to match whole number, fraction and decimal algorithms. They also have opportunities to apply problem-solving skills in drill and practice activities such as games, puzzles, calculator, and computer activities and challenge problems.

4. Geometry and Visualization Skills

Emphasis is placed on "hands-on" activities with both plane and solid shapes. Students estimate and determine perimeter, area and volume. They use a variety of manipulatives to cover the plane and to create designs with pattern and symmetry. They identify properties of common geometric figures, make geometric drawings and constructions, recognize geometric applications in the environment and sharpen visualization skills.

5. Measurement

Emphasis is placed on estimation and other activities which help develop measurement concepts and understandings. Students estimate and then directly measure distance, angles and other quantities to the nearest and smallest unit on the measuring scale. They convert among units of length and weight (mass) within the same measurement system; locate points and give coordinates of points on maps; make scale drawings; determine highway distances from maps; and find measurements, including inaccessible distances, indirectly by using formulas for distance and area.

6. Statistics and Probability

Students predict probable outcomes based on collected data. They construct, read and interpret graphs, tables and charts and make predictions based on them; identify misleading methods of displaying or interpreting data; and determine mean, median and mode using data meaningful to the students.

7. Mathematical Relationships

Students sort and classify geometric figures and sets of numbers and identify number patterns in charts and tables. They use number patterns and relationships to make predictions and solve problems, and make, interpret and explore relationships found in tables of value. Decimal and fraction understandings and 100-grids are used to illustrate percent concepts.

8. Oral and Written Communication

   Students use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

   Students use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade Seven

1. Number and Numeration

   Students demonstrate an understanding of place value to one million, commonly used proper fractions, improper fractions, mixed numbers, decimals, and percents by modeling with concrete and/or pictorial materials and by reading, writing, ordering and comparing. They apply operational properties and recognize and use appropriate mathematical terms, grouping symbols, order of operation and divisibility rules, and concepts related to factoring.

2. Appropriate Computational Skills

   Students have opportunities to select and use all appropriate computational skills including estimation, mental arithmetic, calculators, computers, and pencil/paper. Concrete and pictorial models and "real world" examples are used to demonstrate an understanding of operations with whole numbers, decimals, fractions and percents. Students mentally add, subtract, multiply, and divide whole numbers and decimals by powers of ten and multiples of ten; compute mentally percents such as 25% and 50% and convert mentally between commonly used fractions and decimals. They use a calculator or computer to solve appropriate problems and use rounding and other techniques useful in mental computation to estimate and make approximate whole number, fraction, decimal and percent computations.

3. Problem Solving

   Problem-solving skills such as guess and check, look for a pattern, make a systematic list, make a drawing or model, and solve a simpler related problem are emphasized. Students engage in cooperative problem solving and discuss and compare strategies and solutions to routine and nonroutine problems. They pose new problems, solve multiple step word problems and create story problems to match exercises involving whole numbers, fractions, decimals, and percents. They have opportunities to apply problem-solving skills in drill and practice activities such as games, puzzles, calculator and computer activities, and challenge problems.

4. Geometry and Visualization Skills

   Emphasis is placed on activities and applications with both plane and solid shapes. Students estimate and determine perimeter, area and volume. They use a variety of materials to cover the plane and to create designs with pattern and symmetry. They identify distinguishing properties of common geometric figures, make geometric drawings

and constructions, recognize geometric applications in the environment and sharpen visualization skills.

5. Measurement

Emphasis is placed on estimation and other activities which help develop measurement concepts and understandings. Students select the most appropriate instrument and unit, estimate and then directly measure distance, angles and other quantities to the nearest and smallest unit on the measuring scale. They convert among commonly used units of measurement within the same system; make scale drawings and determine actual distances from scale drawings, blueprints, maps and globes; and find measurements, including inaccessible distances, indirectly by using formulas.

6. Statistics and Probability

Students predict probable outcomes based on collected data. They construct, read and interpret graphs, tables and charts and make predictions based on them; identify misleading methods of displaying or interpreting data; and determine mean, median and mode.

7. Mathematical Relationships

Students use number patterns and relationships to collect, organize, record, and interpret data, to make predictions, and to solve problems. They use models such as 100-grids, number lines or meter sticks to illustrate how percent can be expressed as a fraction or decimal.

8. Oral and Written Communication

Students use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

Students use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade Eight

1. Number and Numeration

Students read, write, order, model and compare commonly used fractions, decimals, percents, and signed numbers. They express large numbers in expanded exponential notation and in scientific notation. They recognize and use operational properties, appropriate mathematical terms, grouping symbols, order of operation and divisibility rules, and concepts related to factoring.

2. Appropriate Computational Skills

Students have opportunities to select and use all appropriate computational skills including estimation, mental arithmetic, calculators, computers, and pencil/paper. Concrete and pictorial models (such as grid paper) and "real world" examples are used to model operations with signed numbers, fractions, decimals, and percent applications. Students mentally add, subtract, multiply, and divide whole numbers and decimals by powers of ten and multiples of ten, and

compute with commonly used percents. They use and apply estimation techniques and check the results of all computations for reasonability.

3. Problem Solving

Problem-solving skills such as guess and check, look for a pattern, make a systematic list, make a drawing or model, and solve a simpler problem are emphasized. Students engage in cooperative problem solving and discuss and compare strategies and solutions to routine and nonroutine problems. They pose new problems, solve multiple step word problems and create story problems to match exercises involving fractions, decimals, percents, and ratio and proportions. They also apply problem-solving skills in drill and practice activities such as games, puzzles, calculator and computer activities, and challenge problems.

4. Geometry and Visualization Skills

Emphasis is placed on activities and applications with both plane and solid shapes. Students estimate and calculate perimeter, area and volume. They use a variety of materials to cover the plane and to create designs with pattern and symmetry. They identify distinguishing properties of common geometric figures, make geometric drawings and constructions, recognize geometric applications in the environment and sharpen visualization skills.

5. Measurement

Emphasis is placed on using and applying measurement concepts. Students estimate and directly measure distances, angles and other quantities and indicate the precision of the measurement. They can demonstrate that answers resulting from calculations with measurements are no more reliable than the measurements used. They make scale drawings and determine actual distances from scale drawings, blueprints, maps and globes; and find and record measurements using proportions and formulas.

6. Statistics and Probability

Students predict probable outcomes based on collected data. They construct, read and interpret graphs, tables and charts and make predictions based on them; identify misleading methods of displaying or interpreting data; and determine, interpret and compare advantages and disadvantages of mean, median and mode.

7. Mathematical Relationships

Students use number patterns and relationships to collect, organize, record, and interpret data, to make predictions, and to solve problems. They interpret and use the concepts of ratio, percent, proportion, and commonly occurring rates.

8. Oral and Written Communication

   Students use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

   Students use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

Grade Eleven

1. Number and Numeration

   Students read, write, order, and compare fractions, decimals, percents, and signed numbers. They express large and small numbers in exponential and scientific notation. Operational properties and order of operation rules are used and applied as are place value and numeration concepts.

2. Appropriate Computational Skills

   Students have opportunities to select and use all appropriate computational skills including estimation, mental arithmetic, calculators, computers, and pencil/paper. Concrete and pictorial models and "real world" examples are used to model operations with decimals, fractions, signed numbers, and percent applications. Students use and apply estimation techniques, compute mentally when appropriate, and convert mentally, manually and electronically among decimals, percents and commonly used fractions. They use calculators and/or computers to solve appropriate problems and recognize the limitations of a calculator when dealing with large numbers.

3. Problem Solving

   Students select and apply the most appropriate tools, methodologies, processes and operations in solving a variety of routine and nonroutine problems. They engage in cooperative problem solving and discuss and compare strategies and solutions. They translate "real world" problems into mathematical statements and mathematical problems and answers back into "real world" context.

4. Geometry and Visualization Skills

   Emphasis is placed on activities and applications with both plane and solid shapes. Students sketch and construct geometric figures, recognize geometric applications in the environment and sharpen visualization skills.

5. Measurement

   Emphasis is placed on using and applying measurement concepts. Students estimate and directly measure length, area, volume, time, and weight (mass) with reasonable accuracy and/or round a measurement to a given unit. They apply ratio and proportion concepts in making and using scale drawings and models and in solving problems; and find and record measurements using proportions and formulas.

6. Statistics and Probability

Students interpret everyday uses of probability and predict probable outcomes based on collected data. They construct, read, and interpret graphs, tables and charts and make predictions based on them. They collect, display and interpret statistical data using mean, median, mode range, and percentile.

7. Mathematical Relationships

Students use number patterns and relationships to collect, organize, record, and interpret data, to make predictions, and to solve problems. They interpret and use the concepts of ratio, percent, proportion, and commonly occurring rates.

8. Oral and Written Communication

Students use and apply the basic communication skills needed to effectively read, discuss and share mathematical ideas and problems.

9. Appropriate Study Skills

Students use and apply study skills which enable them to learn school mathematics effectively and become independent learners.

INSTRUCTIONAL THEMES

Problem Solving

Problem solving is the process of using one's knowledge and experience when encountering new and unexpected situations. It is not a gift of fate, but rather, something that must be taught, modeled, experienced, and reinforced throughout the mathematics curriculum. Problem solving is also an attitude, an approach to thinking about mathematics, and must be nurtured at every stage and in every topic presented in the mathematics program.

Students must understand that problem solving is a process, with solutions coming from exploring situations, brainstorming, and devising and testing strategies over a period of time. They need to develop a positive attitude toward risk-taking and must realize that being temporarily perplexed is a natural and necessary state in problem solving.

Teachers at all levels need to create a classroom atmosphere that is conducive to problem solving. They need to reward students for devising and trying a likely approach even if a solution is not achieved. Students need to feel free to experiment and think original thoughts without the fear of failure. Teachers need to prepare students by teaching them specific problem-solving skills which can be selected and sequenced to form a strategy or solution path. Some skills which can be used include: (1) make a model or diagram, (2) guess, check, and refine, (3) make a systematic list, (4) look for a pattern, (5) work backwards, and (6) solve a simpler but related problem. Next, the teacher must provide students with problem-solving opportunities which are interesting and challenging and yet provide a good chance of success on the part of the students. S(He) must recognize that a challenging problem-solving situation for one group may well be routine for another, and a totally frustrating experience in failure for still another group. Finally, the teacher should assume a facilitator's role as students actively engage themselves in the problem-solving process.

The problem-solving process often includes formulating problems, analyzing the problem, finding solutions, and verifying and interpreting the results. Before one can solve a problem, s(he) must understand the setting, screen relevant and irrelevant information, and identify what is needed to solve the problem. During this first stage it is often helpful for students to work in small groups and share ideas and interpretations of the problem. The teacher should check at this point to make sure that students' interpretations are correct. The second stage, analyzing the problem, is primarily concerned with strategy development. Again, cooperative work in small groups of two, three, or four students can be particularly effective. In groups, students can brainstorm to select and sequence problem-solving skills to form plausible solution strategies. The teacher should monitor this stage and ask leading questions such as "What if . . . ?," "Have you considered . . . ?," "Is that the only way?," but should refrain from giving direct help. In the third stage, students use the data from the problem and apply their strategies to an answer or answers to the problem. This stage is often an individual activity, but it too can be accomplished, cooperatively, in small groups.

The final stage, verifying and communicating results, is a critical component of the problem-solving process. Here students should verify that they have, in fact, solved the problem. They might also explore the possibility that their solution strategy could be extended to a more general problem. Communicating results is a very important culminating or closing activity in which teachers encourage students to share their strategies with the rest of the class. This is important for several reasons: (1) verbalizing the process tends to improve one's problem solving ability, (2) hearing others explain how they solved a problem develops an understanding that there are often several successful strategies for a given problem, and (3) it provides an opportunity for students to be recognized for originality and creative thought.

In summary, problem solving is a process that must be learned through active participation beginning in kindergarten. It is an attitude which teachers need to teach and model at every stage of a student's mathematical development. It calls for a wide variety of instructional formats including direct instruction, facilitating large and small group discussions, and patient guidance while interjecting appropriate hints and answering questions with questions. Problem solving is not magic, but a collection of skills and a willingness to explore ways in which these skills can fit together to form a strategy which leads to a solution of the problem.

Cooperative Learning

In a cooperative learning environment pupils are arranged in small groups, often groups of four, where they work cooperatively toward the solution of a task. Pupils working in small groups have more opportunities to contribute ideas, make speculations, question one another and explain their own thinking. This type of organization is particularly well suited for problem-solving activities because it invites the active participation of each pupil and provides pupils the opportunity to offer ideas and receive immediate feedback.

Cooperative learning improves how students feel about themselves, school, and others. There also is evidence that more pupils achieve at higher levels in cooperative learning situations and that they retain the information longer. Cooperative learning has also been shown to facilitate

- mastery of concepts and principles
- application of information to other settings
- problem-solving skills
- creativity
- verbal skills
- ability to take another person's perspective.

Cooperative learning groups can be used successfully at any grade level. Kindergarten, as well as secondary students benefit from increased opportunity to interact with materials and with other pupils while learning.

Symbolic Thinking*

We use words in dealing with most of our perplexities. However, it is not easy to use words properly in solving problems. Since words are not identical with the things they stand for and since one word may stand for many different things, clear thinking requires the most careful relating of words to things. Mathematicians are especially adept at using abbreviations and written signs. In addition to saving time, a careful selection of symbols assists in the discovery of relationships, e.g., the Hindu-Arabic notation along with the inventions of zero and the decimal point. Mathematics illustrates the gaining of intellectual power and material power over the "real" world through the control of symbols.

Mathematics teachers can help students to develop their own powers of reflective thinking through a more conscious attention to symbols. They can show explicitly how symbols are used in mathematics and frequently make comparisons and contracts with the use of symbols in ordinary language. Three uses of symbols in mathematics are.

1. Symbols such as numbers, units of measure and geometric figures serve as intellectual tools in the conquest of nature. They serve as instruments for describing physical things precisely in terms of size, order, and number.

*Adapted from Progressive Education Association Mathematics in General Education 1938.

2. Symbols are used as a shorthand for other symbols, e.g., a bar graph, "pie" chart, or a curve plotted on coordinate paper are shorthand for tables of values and for relations between data.

3. Symbols are used as symbols for other symbols. Such a use helps free the mind from attention to unnecessary details, e.g., "△" stands for all triangles; a graph is a generalization of a table of particular symbols, the symbols of arithmetic are restricted to illustrative examples but algebra deals with general principles and rules of operation.

Mathematical symbols when compared to non-mathematical symbols are relatively unambiguous and relatively independent of content. They can be substituted with ease for things to which they refer, may be defined once and for all by fiat (zero exponent, i, negative numbers) and are relatively free from emotive variation.

Thinking and Doing Mathematics

Mathematical knowledge is something more than a mere record of the subject matter or the end product of mathematical inquiry. It also implies the "doing" of mathematics. This doing cannot be viewed as an activity individuals engage in by following predetermined rules. In a sense mathematical activity can be seen more as embodying the elements of an art or craft than as a pure technical discipline. Thinking mathematical then implies that learning mathematics is an active process and therefore is not the same as an absorption of the record of mathematical knowledge.

Romberg* identifies four related activities common to all mathematics: abstracting, inventing, proving, and applying. These activities are the natural consequences of mathematical investigations. Students should be exposed to both inductive and deductive reasoning and their roles in problem solving and in the search for relationships. Mathematical instruction also should reflect that concepts can be generalized.

Reading and Writing in Mathematics

In many mathematics classrooms, the only reading done consists of the two-sentence story problems at the end of each chapter. Demands on students' ability to read are not often made, so the ability level is difficult to assess, and little development is done. Regrettably, the real world use of mathematics often involves substantial reading in which information is presented in a form different from that commonly encountered by students in their reading in other classes and decidedly different from the simplistic format of math textbook problems. If students are to meet the goal of becoming prepared to use mathematics, they must gain some exposure to reading mathematics in a practical context.

Coupled with the lack of reading in mathematics courses is the lack of writing in the courses. Answers to problems are "x = -7," or some similar simple expression. Often students are not called on to explain processes, or

*Adapted from Thomas Romberg A Common Curriculum for Mathematics, p 121-159, 1983 Yearbook "National Society for the Study of Education."

challenged by situations which require more than a simple mathematical expression for the answer. This is even more artificial than the little reading done in math courses. This fact is pointed out in several of The Oregon Mathematics Concept Papers, which recommend inclusion of extended problems in the mathematics curriculum, including formal, written reports on these problems.

One example of such an extended assignment which utilizes a variety of mathematical, reading, and written skills could be done in an algebra or geometry course. Students can be given four to eight floor plans for homes with certain similarities (number of rooms, floor space, etc.) but significantly different designs (geodesic dome, Cape Cod, ranch, split level, etc.). They can be assigned to determine what the advantages and disadvantages of each design are, and to prepare a written report in which the data, calculations, and conclusions are presented in an orderly manner. Heating equipment (gas, electric, solar, heat pump), window type, insulation, facing, and other variables can be included. Students can be encouraged to look up the necessary information in the library, in a file in the classroom, or can be given a file folder with the necessary information. In any event, they would do extensive reading in determining the information they need to solve the problem. In doing this reading, they will be evaluating it for applicability and extracting useful information, the basic purpose of much of the reading done in the world of work. Once the information is obtained, it will need to be organized and utilized. Upon solving the problem, the student should show, through a formal written report, the process and conclusions, and justify the conclusions, including any assumptions made.

Extended problems such as discussed above allow the opportunity to integrate mathematics with other disciplines, and use the Essential Learning Skills all together. Though most practical in high school, extended problems can be developed in lower grades as well. Reading, writing, and mathematics do not exist in a vacuum. Only by working with them as an interrelated group can mastery of one, or all be obtained. While extended problems can be very time consuming, shorter "exercises" can be designed which will bring all three disciplines into use. It is essential that such activities be used. Even the most gifted of our current mathematics students are frequently unable to use their mathematics in "real world" situations because of their inability to read and write mathematics and the information which leads to or flows from the mathematics. If we do not demand reading and writing in the mathematics classroom, we will be preparing students to do mathematics, but not to be mathematically literate for the "real world" in which mathematics is applied.

Appropriate Computational Skills

Computation is the execution of a chosen strategy to solve a quantitative problem. The mode of execution may include any or all of the following: (1) mental calculation, (2) paper and pencil figuring, (3) calculator, and (4) computer. Determining either precise or approximate results is common when using the first two modes. The operations involved in execution include addition, subtraction, multiplication and division of whole numbers, decimals, fractions, percents, and integers.

When we observe computation being used in society, we see a variety of computational skills being used to solve problems. Most nonjob related computation is done mentally, much of it approximate calculations. One study found that nonjob related computation is about 70 percent mental and 25 percent calculator-assisted. On the job, most computation (up to 90 percent) is technologically assisted (calculator or computer).

There appears to be a marked discrepancy between the types of computation being taught in school and the actual needs of pupils outside the classroom. An examination of basal texts reveals that most deal primarily with paper and pencil calculations. One study of classroom practices found that 90 percent of instruction involved paper and pencil algorithms. If mathematics educators are to meet the actual computational needs of students outside the classroom we must:

1. Recognize the need to provide instruction in the several modes of computation and approach that instruction in such a way that students gain skill in making the decision regarding which mode(s) is (are) appropriate for each problem in which computation is needed.

2. Recognize the need to provide instruction in both approximate and precise calculations and in helping students decide which is more appropriate for each problem which includes computation.

In light of the varied computational needs of today's students, it is time to recognize that computation is not an end. Rather, it is just one part of the problem-solving process--one problem-solving tool. Hence, teaching the several modes of computation is an effort to place computation into context--a means to an end.

Calculators and Computers

Since less than 1 percent of college freshmen plan on majoring in mathematics*, it seems obvious that the primary goal of math education is to prepare students to cope with the many problems they may encounter on the job, in future education, and in their day-to-day activities. Keeping this in mind will provide some measuring stick for determining the appropriate use of calculators and computers in the mathematics classroom.

It is reasonable, when introducing new concepts, to use calculators to do problems as examples that are more complex than those already done. It is obviously not appropriate, however, to allow students to rely on electronic devices to carry out simple computations before they understand the processes. Once understanding has been achieved, the advisability of using pencil and paper techniques rather than calculators for solving many problems is questionable. Applying the criteria of future use, even a casual survey of those around us who are routinely involved in "number crunching" will reveal that few instances arise where pencil and paper operations are performed. The normal process will involve either calculator, or for extremely involved or repetitive operations, computers. Continued emphasis on pencil and paper computation is not consistent with this reality and serves primarily to test the students' tolerance for drudgery, not their mastery of topics. These considerations should apply in most testing situations as well as during instructional activities.

Characteristically, the American educational system has emphasized computation more heavily than the European nations and has devoted more time to it. Both the National Council of Supervisors of Mathematics* and the National Council of Teachers of Mathematics** have issued position papers in which the importance of other content strands has been emphasized and the importance of computation diminished. The NCTM in particular has emphasized the role which calculators*** can play in shifting this emphasis. If the mathematics curriculum in the primary, middle, and secondary grades is to reflect the recommended increase in time devoted to other aspects of mathematics, particularly problem solving, time must be gained by the use of calculators in place of pencil and paper computation once concepts have been mastered.

*Astin, et al., The American Freshman: National Norms for Fall 1980, as reported in the Conference Board of the Mathematical Sciences Report of the Survey Committee, Volume VI, 1981.

*National Council of Supervisors of Mathematics, "Position Paper on Basic Mathematical Skills," The Mathematics Teacher 71 (February 1978: 147-152)

**National Council of Teachers of Mathematics, An Agenda for Action: Recommendations for School Mathematics of the 1980s, 1980

**_____ The Impact of Computing Technology on School Mathematics

***_____ A Position Statement on Calculators in the Classroom, September 1985.

Finally, some aspects of other content strands can be explored conveniently only through the use of calculators.

Calculators, therefore, become an essential part of a forward-looking mathematics curriculum. Students should be expected to obtain calculators, or access to calculators should be provided for them. Certainly, by the third grade, calculators should be considered an essential tool for doing mathematics. In the middle school and high school courses, students should be allowed, if not encouraged, to use calculators in math classes. The logic of continuing to have Algebra I and Algebra II students perform calculations involved in solution of complex or involved equations is questionable. The concepts being learned are not computation concepts. Improved accuracy, and thus a decrease in frustration, can be obtained by the use of calculators.

While calculators are now within virtually everyone's reach, and are being used increasingly frequently, computers have made less impact in the mathematics classroom. With the exception of computer literacy and programming courses, and the use of relatively simple programs designed for CAI, primarily in the elementary grades, computers have not become "tools" used in the many mathematics classrooms.

As more sophisticated software is developed, computers will become more common. CAI programs for most of the standard secondary mathematics courses are now available, and steadily improving. Specific application of software, performing such tasks as solving first and second order equations, factoring, graphing, differentiation, and integration are also appearing. Examining "real world" use of such programs, it seems likely that their use in nonschool professional environments will increase. This suggests that there use in the classroom should also increase. The same criteria previously applied to calculators, namely, is the task central to the concept being learned or merely a time-consuming task previously mastered, should be applied. Where the task is not central, students should be encouraged to use a computer. Obviously, this suggests that school districts will need to provide increased budgets for the software and hardware needs of the mathematics curriculum.

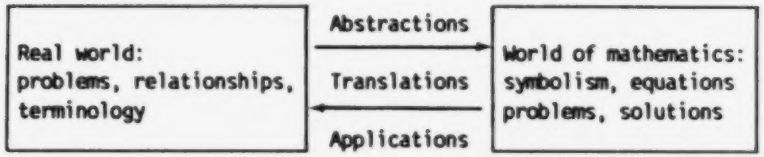
Applications

According to Zalman Usiskin*, "Skill is relatively useless without the ability to apply the skill; all courses should devote some attention to 'real world' applications of the mathematics being studied. In the college-bound stream, applications are often not included in the curriculum. Teachers often skip applications even when they are in the book." If we agree with the idea that the primary purpose of the mathematics curriculum is to prepare students for its use later in life, application must be considered at least a vital as Usiskin maintains.

Applications should be included in all units in the mathematics curriculum. The selected applications should be consistent with students' level of development and perceived by them as real and relevant to their world. This simplified diagram suggests the relationship that

*Usiskin, Zalman, A Proposal of Re-Forming the Secondary School Mathematics Curriculum, NCTM Annual Meeting, 1983.

should exist between mathematics instruction and the "real world":



Notice that the "real world" influences mathematics and mathematics influences the "real world." Thus, mathematics instruction must reflect not only aspects of the world and mathematics but of their interactions.*

Thus, in doing applications in mathematics we are concerned with developing relational thinking. The concept of relation is fundamental to human thought. Knowledge of relationships allows humans to go beyond the immediate and make reasonably reliable predictions of future events.** Scientists and engineers use applications of mathematics to explain physical phenomena and to control them. "It is in precisely this fundamental problem of searching for relationships and giving accurate expression to them that mathematics has been so successful."**

*Schaaf, Oscar et al., Oregon Mathematics Concept Paper No. 2: Middle School mathematics, 6-8, 1986.

**National Council of Teachers of Mathematics, The Place of Mathematics in Secondary Education, NCTM 15th Yearbook.

A knowledge of mathematical relationships is necessary, but more important still is the development of the ability to search for relationships both in mathematics and everyday situations. Students must learn to "translate" between mathematics and practical situations. This process must go both ways. The ability of relational thinking is more likely to develop if the instructional focus is as much on the search process as upon the relationships themselves.

The reality as seen in the mathematics classroom, however, is that application plays a minor role in instruction if it is present at all. The reasons for this are involved and complex, but can be broken down into three basic justifications. The first, often quoted by frustrated teachers, is that "Students simply don't seem to be able to do story (application) problems." To avoid frustrating students, which results in low grades, large numbers of students dropping courses, and declining registration in the courses, and to minimize their own frustration, teachers often don't even attempt to teach applications. The second is that in most texts the "applications" appear in the form of "word" or "story" problems. The relevance and reasonableness of these problems, or rather the lack of these qualities is a great source of amusement to students and teachers alike. A parody of such problems (If a chicken and a half can lay an egg and a half in a day and a half, how long will it take a grasshopper with one wooden leg to kick all the seeds out of a Dill pickle?) has even been turned into a greeting card.

The third, and perhaps the most compelling justification teachers give for not doing applications is lack of time. The huge volume of material teachers feel they must cover

intimidates them into giving up the attempt to teach applications. This reason, however, is the most obviously incorrect. The importance of applications is far greater than the importance of completing all topics in our textbooks. Careful analysis of textbooks will allow experienced teachers to eliminate topics, freeing time for application, possibly the most important skill taught in the math curriculum. Appropriate use of calculators and computers to eliminate some computational drudgery can free more time.

The first reason, students' inability to do applications, is also easily countered. If this idea were extended, most students would be excused from most of the math, science, and English curriculum.

Lack of materials is an equally weak argument. In the area of algebra, Usiskin's two-volume algebra text* provides an excellent sourcebook for those using other texts, even if not used as a primary text. Problems for general mathematics, algebra, geometry, and other courses are available from a number of different publishers** and are discussed in more detail in the bibliography of the OMEC paper, Vocational and Technical Mathematics*. These problems can give students practice in applying the various mathematics content taught, while at the same time demonstrating the usefulness of that content and developing skills in the process of "translation." In particular, they lend themselves to developing extended problems which use a number of concepts and applications.

Even without the various sources already referred to, a number of applications can be developed by using material readily available to math teachers from their colleagues in their science departments. A simple series of electrical circuit opens up the possibility of exploring linear and inverse relationships. Exploration of Ohm's Law through a demonstration or experiment can illustrate the use of standard and log-Olog graph paper in determining the relationship between two variables. Doing a Hooke's Law demonstration with a spring can allow students to explore situations in which equations apply over one range of values, but do not apply over all values, due to physical considerations. In this way they begin to appreciate the limitations of mathematics as applied to real situations. A more unconventional approach, but an equally good one, would be to first consider a physical problem, then use it as motivation for developing the appropriate mathematics.

*Usiskin, Zalman, Algebra Through Applications NCTM 1979.

**Delmar Publishers, Practical Problems in Mathematics for eleven different volumes dealing with different voc/tech areas.

National Council of Teachers of Mathematics, Applications in School Mathematics - 1981 Yearbook.

Oregon Department of Education, Oregon Vo-Tech Mathematics Problem Sets, 1974.

Wiltsie, David, Vocational Mathematics Series, Motivation Development Inc., 1982.

*Zaraza, et al., Oregon Mathematics Concept Paper No. 7: Vocational and Technical Mathematics, 1986.

STAFF DEVELOPMENT

The previous section of this document - Instructional Themes - suggested a variety of staff development needs. The following checklist can be used as a tool in assisting administrators and teachers in identifying those needs and suggesting a variety of staff development approaches. When was the last time:

<u>as a classroom teacher, you</u>

_____ created a worksheet for your students?

_____ really felt good about a math lesson?

_____ used an interesting approach to skill building?

_____ shared one of your ideas with another teacher?

_____ found a good idea when visiting another teacher's classroom?

_____ asked the librarian to order a math book for your students or for your own professional growth?

_____ had students solve a problem using things other than paper and pencil or textbooks?

_____ took or helped plan a workshop or inservice?

_____ asked for released time to do any of the above?

<u>as an administrator, you</u>

_____ attended a math lesson?

_____ taught students a math lesson?

_____ helped teachers plan inservice meetings or workshops in math?

_____ encouraged your teachers to visit another math class?

_____ utilized the skills of your teachers having special math interests?

_____ helped a teacher order math materials other than textbooks?

_____ recommended a resource center for one of your teachers to visit?

_____ helped teachers take advantage of released time to develop these interests?

No textbook can have the impact on student learning that a knowledgeable well-trained caring teacher can. However, some teachers continue to teach mathematics in the same way that they learned mathematics: memorizing a tedious sequence of algorithms and definitions.

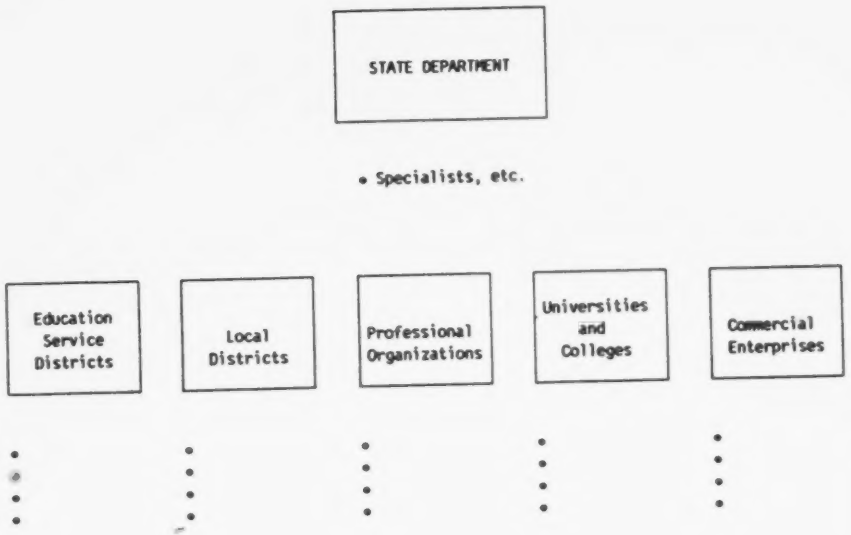
Mathematics must be viewed differently. The teacher must exhibit an attitude of exploration and invention, conveying the idea that all students can learn, enjoy, and use mathematics.

School districts should provide professional growth experiences for their teachers. Teachers who have used only a textbook approach must have inservice training to develop skills which enable them to provide more versatile instruction. Districts must provide ongoing financial support, encouragement, and time for teachers to attend conferences and workshops.

Teachers and buildings should be encouraged to maintain current membership in professional organizations to insure access to current trends and research. Teachers should be encouraged to actively participate in professional organizations.

Given the new suggested curriculum and required learning skills, districts which do not provide an ongoing staff development program will be unable to fully reach the mathematics goals described in this guide.

GROWTH OPPORTUNITIES



ORGANIZATIONAL PATTERNS AND TEACHING CONSIDERATIONS

Role of the Textbook

Ideally, the mathematics textbook should be regarded as a guide to the specific subject area. In general, it provides far more material than can be reasonably covered in a single year. By doing so, it provides a number of options to the teacher, presenting a variety of somewhat different courses depending on which topics are used and which are omitted. At the same time major topic strands recommended by the Oregon Council of Teachers of Mathematics (OCTM), the National Council of Teachers of Mathematics (NCTM), and the Oregon Mathematics Education Council (OMEC) are absent.

In reality, the textbook is more frequently used as a "cookbook." Too often teachers begin on page 1 and proceed as far as they can in the instructional time allotted. In doing so, they often cover material that some experienced teachers would regard as unnecessary. The first chapter(s) is (are) often review, yet may be taught with the same depth and time commitment as new material of equal length. Later, more important chapters, may never be reached.

This problem occurs because teachers appear to assume that the authors know exactly what should be taught and have included all essential topics. Additionally, they mistakenly believe all the material must be presented in order, that no topics are of lesser importance and could be deleted or deemphasized. They are unaware that an excess of material is presented to "cover all the bases," to insure that potential purchasers will find what they regard as important.

Current TSPC standards for mathematics certification insure that math teachers have an adequate, broad exposure to and understanding of mathematics. Such teachers are better prepared to pick and choose topics from texts. Through consultation with colleagues and consideration of journal articles, reports and position papers which discuss appropriate content for the various courses, such as Usiskin's*, they can make reasonable choices regarding the content of their courses. They should not be rigidly bound by the pace, arrangement, or content of a text- book. The textbook should merely serve as a starting point, and perhaps a source of problems and exercises. Only if well prepared teachers are encouraged to exercise their professional judgment can change and improvement be implemented at a reasonable pace.

In choosing supplemental materials, useful sources should include books, newspapers, magazines, project materials, maps, games, and computer software. Teachers should feel free to make extensive use of materials outside the standard textbook. In this way they can best tailor courses to meet the specific needs of their students.

*Usiskin, Zalman. "What Should NOT Be in the Algebra and Geometry Curricula of Average College-Bound Students?" Mathematics Teacher September 1980: 413-424.

Providing for Individual Differences/Remediation

A common problem which many Americans suffer from is "math anxiety." The implication of this "ailment" is that there is something wrong with the individual who suffers from it. S(He) is somehow flawed because s(he) cannot learn mathematics as well or as quickly as the norm, or the average. When this norm is examined, or when the withdrawal and flunking rates in mathematics courses are reviewed, it becomes obvious that most of us do not achieve at the "norm." The conventional mathematics curriculum has been geared toward a single goal: calculus. In such programs, from primary mathematics through high school the pace and content of math courses are designed to prepare students for calculus. Any student who fails to keep up the pace is viewed as "having a problem" with mathematics.

This is a misperception. The student does not have the problem. The curriculum does. Our conventional mathematics curriculum tends to lack the flexibility which allows students to reach their own maximum ability. Instead, it serves to discourage those who cannot achieve at a standard pace. A recent survey indicated that fewer than one percent of graduating high school students have taken calculus. Even in college, fewer than 20 percent of students major in fields which regularly require or use calculus. This means most students "drop out" of mathematics along the way.

Rather than designing the curriculum to fit the needs of students, we design the curriculum, then force students to fit into it or leave mathematics. More emphasis on individual needs is necessary.

One level at which an awareness of individual differences and attempts to accommodate these differences has been successful is the primary mathematics program. Primary teachers routinely ability group students across the various content areas. This grouping allows students to progress at a more comfortable rate. The difference in achievement that occurs at these grade levels, however, helps generate problems in later grades. During middle school, and continuing throughout high school, allowance should be made for these increasing individual differences.

If students are to be encouraged to continue in mathematics, if they are to become mathematically literate, appropriate learning opportunities must be provided. The two-year graduation requirement provides both an opportunity and an obligation to develop a curriculum which allows for differences in ability and interest in mathematics. Courses which are not explicitly preparation for calculus, but rather oriented toward application, or toward general mathematical literacy (including the broad content strands recommended by the NCTM, NCSM and the OMEC concept papers) must be developed. The curriculum must be broadened to allow more options for students. This must take place not only at the low end, where inclusion of all the recommended content strands can provide a more useful and interesting course, but at the upper end, as well, so that students who have been successful have an option other than calculus.

Organizing Systems for Secondary

Historically, the major system or vehicle by which mathematic instruction has been delivered has been through a sequential system of course offerings. An alternative approach, one in which mathematics is taught as a "unified" discipline rather than as a neatly compartmentalized series of discrete topic areas, is more common in the European school systems. The idea has received substantial support from the mathematics community in this country in the past. The Mathematics Association of America (MAA), in 1923, recommended a program of "composite," "correlated," "unified," or "general" courses to be introduced into secondary schools so that students might grasp the important interrelationships between arithmetic, algebra, and intuitive geometry in order to gain insight into mathematical methods.* The 1940 report of the Joint Commission of MAA and NCTM also advocated a unified approach as its first option.** The Oregon Mathematics Concept Papers, developed by OMEC, frequently refer to position papers of both the NCTM*** and the National Council of

*Mathematics Association of America, The Reorganization of Mathematics in Secondary Education, 1923.

**Joint Commission of MAA and NCTM, The Place of Mathematics in Secondary Education Fifteenth Yearbook, 1940.

***National Council of Teachers of Mathematics, An Agenda for Action: Recommendations for School Mathematics of the 1980s, 1980.

Supervisors of Mathematics (NCSM)* which advocate inclusion of a variety of strands in all mathematics courses. Two of these strands are topic areas which are normally separate courses in the traditional sequential presentation of mathematics.

In view of the historic support for the unified approach to mathematics instruction and the more recent advocacy of such ideas by the professional mathematics teaching organizations, the unified approach must be regarded as a preferred method of presentation.

In the past, suggesting that the unified approach be adopted was somewhat impractical. Few materials were available for such approaches. Recently, however, there has been a trend in that direction which promises to motivate text authors and publishers to develop a greater array of materials, making this option not only viable, but desirable for forward-looking school districts. (At the time this paper was written, three publishers already offered complete sets of textbooks utilizing a unified program.) New York state has a unified program for those students planning to take the Regent's examination before graduation. The Superintendent of Public Instruction for the State of Washington has recommended** three

*National Council of Supervisors of Mathematics, "Position Paper on Basic Mathematical Skills," The Mathematics Teacher, 71, (February 1978), pp. 147-152.

**Guidelines for the 9-12 Mathematics Curriculum from the Office of the Superintendent of Public Instruction from the State of Washington.

sequences of courses for the unified study of mathematics, each designed for students of differing ability, interests, and aspirations. The recently published Mathematics Framework for California Public Schools K-12 mandates that concepts from the various content strands be incorporated into mathematics courses at all grade levels.

Based on the positions taken by mathematics educators, both nationally and within Oregon, the unified approach to the mathematics curriculum must be regarded as the wave of the future. As a means of adequately serving the needs of all students, rather than merely the college-prep, such a program is far superior to the more traditional or sequential approach. It allows all students to progress beyond mere computation, opening up a world of interrelated mathematics. Additionally, it allows more flexibility for different rates of student development, diminishing the "selecting out" process common to sequential programs in which a student, placed in a class such as formal geometry before they have developed the necessary reasoning ability, is frequently lost to mathematics forever. School districts are strongly encouraged to explore the possibility of implementing a unified approach to mathematics as they review their mathematics curriculum. The OMEC concept paper "An Integrated Mathematics Curriculum for High School" presents additional arguments supporting this option, as well as outlining a possible approach in more detail.

Emphasis on delivery of instruction techniques in the unified approach is often more heavily emphasized than in the sequential curriculum. If the fully unified curriculum cannot be implemented by school districts, serious attention should at least be paid to some of these key instructional methodologies:

1. Students should be involved in active learning situations for at least half of the average class period.

2. Individual seat work should not exceed one-third of the class period.

3. Oral and written description of problem solving processes should be required of students.

4. Cooperative learning should be an essential element of the learning process in the mathematics classroom.

5. Teacher modeling of problem solving is essential.

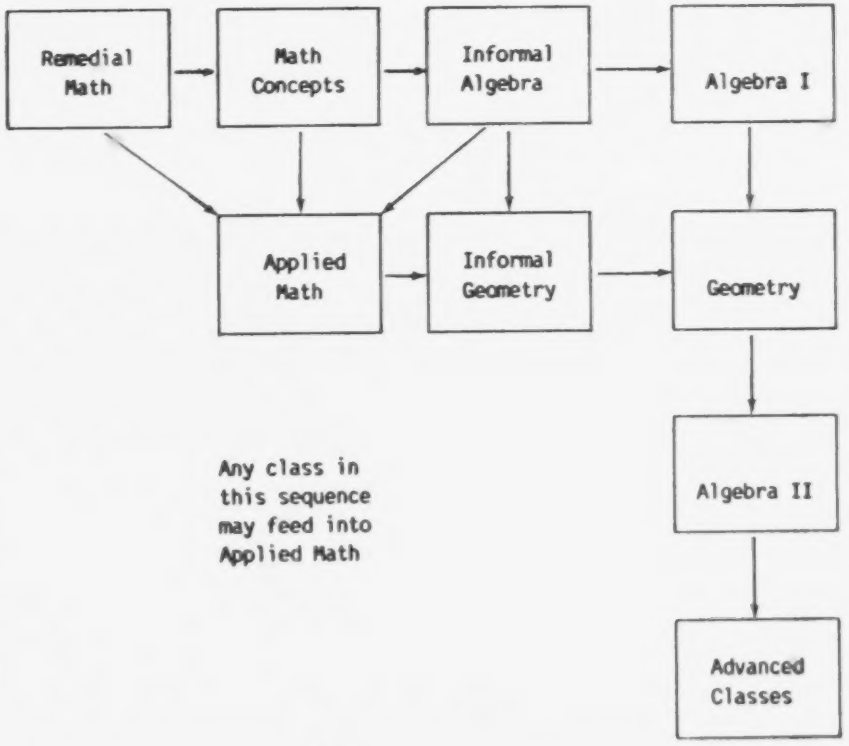
6. Mathematics should be presented as a sequential discipline (not purely within algebra, for example, but in the context of the whole of mathematics).

7. Questioning, by both students and instructor, must be encouraged. Straight lectures must be minimized.

8. Calculators and computers should be employed to eliminate the tedium of lengthy calculations, to allow topics to be developed in greater detail and to allow more realistic problem solving.

9. Extended or in-depth problems (of several days in length) should be included to allow students to develop problem solving, analytical, and reporting skills in a more realistic environment.

Those school districts which choose to continue to utilize a traditional sequential curriculum are faced with the necessity for substantial modification, particularly with the two-year graduation requirement. The Algebra I - Geometry - Algebra II - Advanced Topics sequence so commonly used is actually designed for the college-bound, math and science oriented students, a small percentage of all those students actually subjected to it. To truly fill the needs of students, alternatives at all levels must be considered. A possible program to achieve this is shown below:



All of these courses should emphasize more use of calculators than is presently the common practice. They follow the same content strands which are being recommended for the middle school mathematics (which is a unified program).

Remedial Math. This course is designated for students ranging from special education students through students who are no more than two or three years below grade level. It is assumed that these students have not yet have mastered the basic concepts and operations of arithmetic.

Math Concepts. For students no more than two to three years below age/grade level, this course should follow the curriculum included in The Oregon Mathematics Concept Papers on the middle school curriculum, or alternatively, Zalman Usiskin's Transition Mathematics course. The emphasis here should **not** be on computation. Course content should follow the strands referred to throughout the discussion of the K-8 curriculum.

Applied Math. This course may be a single course or several different courses, depending on the size and needs of the school. Two of The Oregon Mathematics Concept Papers, "Oregon Graduation Requirements: Mathematics" and "Vocational and Technical Mathematics," provide a number of possible curricula. Emphasis should be on the use of calculators and on "real-world" problem solving.

Informal Algebra. This course and the course on informal geometry should not follow the often repeated pattern of the introductory algebra course--slower presentation but the same material. This should not follow the "slower but louder" teaching approach used with less

accelerated students. Certain traditional concepts in algebra should be omitted completely, or done only through the use of computer programs designed to carry out the operations. These include quadratic equations, factoring, and simultaneous solution of equations by substitution. The course should build a conceptual base, minimizing an algorithmic approach. Equations should be solved intuitively when possible, with models used as well. The orientation must be practical. Problems should provide the motivation for the development of the algebraic concepts. The various mathematical properties should be examined and developed, but not emphasized by name. Drawing and graphic interpretation (particularly with the aid of the computer) should be utilized wherever practical.

Informal Geometry. This course must de-emphasize proof. Primary focus should be on construction, measurement, drawing, categorization and classification, property identification, and graphic interpretation.

Algebra I - Geometry - Algebra II. These courses should include more integration of the various content strands than is presently the case.

Advanced Courses. Current practice generally offers senior math, pre-calculus, or calculus courses in this slot. While these may still be options, serious consideration should be given to placing discrete mathematics, probability and statistics, or problem solving courses at this point in the curriculum.

While this curriculum is not as desirable as the fully unified program, it does provide for a wide variety of students. Advanced students may, of course, enter at high levels, such as Geometry, Algebra II, or even the advanced course.

Teaching Considerations

At the high school level, there is a wide range of student ability, skill, and background. Thus, the content varies widely. It is true, however, that the delivery of instruction can be very similar for the majority of the high school population.

Instruction in the high school mathematics program should:

- Involve students in active learning situations. Individual seatwork should be minimized.
- Require that students describe in writing and orally the process used in solving a given problem and the results obtained.
- Involve questioning techniques that create student interaction, encourage exploration, diagnose processes that students have used, enhance transfer of learning, motivate, and challenge students to think.
- Stimulate curiosity on the part of the students and set the stage so that students ask questions and contribute comments in a discussion.
- Include the model of problem-solving activities by the teacher as a regular part of the instructional program.

- Demonstrate the continuity of mathematics by explaining the relationship of the current work to concepts learned in the past and to be learned in the future.

- Use a variety of instructional formats, e.g., small groups, interactive groups, teacher-led discussions, student presentations, and laboratory learning experiences.

- Involve high but appropriate expectations for all students, for example, students are expected to complete all assignments and to do well on examinations.

- Involve the use of calculators and computers to: (1) enhance the study of mathematical relationships, and (2) speed up the development of mathematical concepts by reducing the time spent on intermediate computations.

- Encourage students to solve problems in a variety of ways and to accept solutions in many different forms.

- Provide opportunities for students to become involved with in-depth problem situations for periods of several days. As they work toward solutions, students should investigate and experiment, develop new mathematical skills and concepts, apply previously acquired knowledge, and be able to express the solution in written form.

- Increase the student's ability to estimate solutions and to always judge the reasonableness of calculated results.

Alternative Approaches to Two-Year Graduation Requirement

With the adoption of the two-year graduation requirement in mathematics, the Oregon Board of Education has created an opportunity for the development of a relevant and effective mathematics component for all students. A survey of Oregon high schools taken by the OMEC Subcommittee on Vocational and Technical Mathematics* indicated that 20 percent of the graduating class of 1984 took only one year of mathematics. That same survey revealed that 35.8 percent of graduates took no math beyond Algebra I and 51.2 percent did not go beyond Geometry. This indicates that a large number of Oregon students have been choosing to not take courses in the traditional mathematics curriculum. They present a large "clientele" which has not always been well served but now must be served.

For many of the high school students who will be taking the minimum mathematics requirement, the traditional sequences are unsatisfactory. At the lowest achievement level, students have normally been placed in classes which emphasize computation, a content strand in which these students have already demonstrated they have little facility. At a slightly higher level, students are placed in courses which are essentially part of the algebra sequence which has been designed as a "feeder" for Calculus. While algebra may be a suitable course for the calculus-bound student, it does not serve the needs of students for whom it will be the last course in mathematics.

*Zaraza et al., Report to the Oregon Mathematics Education Council on Vocational and Technical Mathematics, 1985.

Either of the traditional sequences taken by students taking minimal mathematics emphasize content which is often not useful to them. Appropriate courses for these students should focus on mental computation, application and use of simple algebraic formulate, interpretation of graphs and statistics, information geometry, measurement, and problem solving*. Mastery of these concepts will enable these students to cope more effectively with the adult world.

Accordingly, courses should be designed for these students which address content other than simple computation or algebra. A number of options for such courses are outlined in the OMEC concept papers on the two-year requirement and on voc/tech mathematics. Both of these papers emphasize that courses designed to fulfill the two-year requirement should follow the NCTM and OCTM recommendations that all content strands of mathematics be developed. For these students, this is a radically different approach, but a more realistic one.

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*Hopper et al., Graduation Requirements Committee Report, OMEC 1985.

Another student population which must be considered is that group which completes Algebra I or its equivalent in its first year of high school, but historically has not taken additional math. Relatively poor performance in algebra, fear or dislike of geometry, or a general fear of mathematics or not recognizing its importance in our current technological society are the major reasons for not taking additional mathematics. With the two-year requirement, these students are now placed in the position of having to take additional math courses. Again, the option of offering nontraditional mathematics courses such as those suggested in the OMEC concept papers seems to be a viable approach. Such courses, when properly designed can provide interesting and useful mathematics for a far broader ability range of students than the more conventional courses.

1.0 Number and Numeration: Students demonstrate an understanding of number and numeration concepts and use these understandings to interpret and solve problems.

1.1 READ, WRITE, ORDER, COMPARE AND USE NUMBERS (ELS 1.4)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Rote count to 20 and demonstrate comprehention of whole numbers up to 10 by reading, writing, ordering, modeling, and comparing	a Rote count to 20 and demonstrate comprehention of whole numbers up to 20 by reading, writing, ordering, modeling, and comparing	a Rote count to 100 and demonstrate comprehention of whole numbers up to 100 by reading, writing, ordering, modeling, and comparing	a Read and write whole numbers to 1,000, commonly used fractions (1/2, 1/3, 5/10), and decimals in tenths and hundredths using money models	a Read and write whole numbers to 100,000; model commonly used proper fractions; mixed numbers and improper fractions (1 1/2, 2 1/4, etc.)
b Use number patterns to facilitate skip counting by 5 and 10	b Demonstrate the counting skills of skip counting by 2, 5, and 10; "counting on" and counting backwards to and from 20	b Demonstrate the counting skills of skip counting by 2, 5, and 10; "counting on" and counting backwards to and from 20	b Demonstrate the counting skills of skip counting by 2, 5, and 10; "counting on" and counting backwards to and from 100	b Demonstrate skip counting skills
c Estimate number of objects and check reasonableness of answers by counting	c Estimate number of objects and check reasonableness of answers by counting	c Estimate number of objects and check reasonableness of answers by counting	c Use place value concepts to estimate and sense reasonableness of answers	c Use place value concepts to estimate and sense reasonableness of answers
d Use ordinals (1st, 2nd, 3rd, 4th, and 5th)	d Use ordinals to 10th			

* The outcomes for mathematics in the skills and grade level expectancies that are underlined reflect the required outcomes from the Common Curriculum Goals and includes the Essential Learning Skills.

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Read and write whole numbers to one million; commonly used proper fractions, mixed numbers, and improper fractions; and decimals to thousandths	a Read and write whole numbers to one million; commonly used proper fractions, mixed numbers, and improper fractions; and decimals to thousandths	a Read and write numbers, including decimals, commonly used fractions and percents	a Read and write numbers, including decimals, commonly used fractions and percents	a Read, write and order numbers including decimals, commonly used fractions, percents and numbers in scientific notation
Demonstrate skip counting skills	b Write numbers in expanded notation (e.g., 623 = 600+20+3)	b Write numbers in expanded notation (e.g., 623 = 600+20+3)	b Express large numbers in expanded exponential notation	b Express large and small numbers in expanded exponential notation
c Use place value concepts to estimate and sense reasonableness of answers			c Order signed numbers and commonly used fractions, decimals and percents	c Order signed numbers and commonly used fractions, decimals and percents
			d Express large whole numbers in scientific notation	d Express large and small numbers in scientific notation

1.2 USE CONCRETE AND PICTORIAL MODELS TO DEMONSTRATE NUMBER AND NUMERATION CONCEPTS (ELS 1.4)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Use concrete models to show place value and to demonstrate halves and wholes	a Use concrete models to show place value through grouping and exchanging, and to demonstrate halves and wholes	a Use concrete models to demonstrate two-digit place value through grouping and exchanging, and to demonstrate halves, wholes, thirds, and fourths	a Order, compare and model (demonstrate comprehension by use of objects or a drawing) place values to 1000, commonly used fractions and decimals (using money models) in tenths and hundredths	a Order, compare and model place values to one million; commonly used fractions and decimals to hundredths, and use concrete and pictorial models to demonstrate an understanding of the above
		b Identify the number of ones and tens in numbers less than 100 and use concrete models to demonstrate understanding	b Identify the number of ones, tens and hundreds in numbers less than 1000 and use concrete models to demonstrate understanding	b Identify the number of ones, tens, hundreds, thousands, and ten thousands in numbers less than 100,000 and use concrete models to demonstrate understanding

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Order, compare and model place values to one million; commonly used fractions and decimals to thousandths, and use concrete pictorial models to demonstrate an understanding of the above	a Order, compare and model place values to one million; commonly used fractions and decimals, and use concrete and pictorial models to demonstrate an understanding of the above	a Order, compare and model commonly used fractions, decimals and percents, and use concrete and pictorial models to demonstrate an understanding of the above	a Order, compare and model commonly used fractions, decimals, percents, and signed numbers, and give examples of positive and negative quantities (e.g., temperature, football, bank balances, altitude)	a Order, compare and model commonly used fractions, decimals, percents, and signed numbers, and give examples of positive and negative quantities (e.g., temperature, football, bank balances, altitude)
b Identify the number of ones, tens, hundreds, thousands, ten-thousands, and hundred-thousands in numbers less than one million, and tenths, hundredths, thousandths in numbers less than one	b Identify the number of ones, tens, hundreds, thousands, ten-thousands, and hundred-thousands in numbers less than one million, and tenths, hundredths, thousandths, etc., in numbers less than one	b Identify the number of ones, tens, hundreds, thousands, ten-thousands, and hundred-thousands in numbers less than one million, and tenths, hundredths, thousandths, etc., in numbers less than one	b Identify the number of ones, tens, hundreds, thousands, ten-thousands, and hundred-thousands in numbers less than one million, and tenths, hundredths, and thousandths in numbers less than one	b Identify the number of ones, tens, hundreds, thousands, ten-thousands, and hundred-thousands in numbers less than one million, and tenths, hundredths, and thousandths in numbers less than one
	c Describe the usefulness of a place value numeration system in mental computation giving examples in common use	c Describe the usefulness of a place value numeration system in mental computation giving examples in common use	c Describe the usefulness of a place value numeration system in mental computation giving examples in common use	c Describe the usefulness of a place value numeration system in mental computation giving examples in common use
		d Compare the base ten numeration system with an ancient system	d Explain societal uses of modern, nondecimal systems of numeration	d Explain societal uses of modern, nondecimal systems of numeration, particularly binary
			e Illustrate the meaning of square root by use of a model (e.g., in a square the side represents the square root of the area)	e Illustrate the meaning of square root by use of a model (e.g., in a square the side represents the square root of the area)

1.3 RECOGNIZE AND USE NUMBER PROPERTY CONCEPTS

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
	[a] Demonstrate the properties of addition and subtraction	[a] Demonstrate and use the properties of addition and subtraction	[a] Demonstrate, use and apply the properties of addition, subtraction and single digit multiplication	[a] Demonstrate, use and apply the properties of addition, subtraction, multiplication, and single digit division with whole numbers
	[b] Recognize and use mathematical terms (e.g., equal, greater, less)	[b] Recognize and use mathematical terms (e.g., odd, even, equal, sum, and difference)	[b] Recognize and use mathematical terms (e.g., sum, total, difference, product, equal, odd, even)	[b] Recognize and use mathematical terms (e.g., product, factor, quotient, remainder, sum)
				[d] Recognize and use concepts related to multiples and factors

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Demonstrate, use and apply the properties of addition, subtraction, multiplication, and division with whole numbers	a Use and apply the properties of addition, subtraction, multiplication, and division with whole numbers	a Use and apply the properties of addition, subtraction, multiplication, and division with whole numbers, positive rational numbers, and demonstrate properties of one and zero (including nondivision by zero) and the closure property	a Use and apply the properties of addition, subtraction, multiplication, and division with whole numbers and positive rational numbers, and demonstrate properties of one and zero (including nondivision by zero) and the closure property	a Use and apply operational properties
b Recognize and use mathematical terms (e.g., product, factor, quotient, remainder, sum)	b Recognize and use mathematical terms	b Recognize and use mathematical terms	b Recognize and use mathematical terms	b Recognize and use mathematical terms
	c Explain the reasons for the rules for order of operations and use of grouping symbols	c Explain the reasons for the rules for order of operations and use of grouping symbols	c Explain the reasons for the rules for order of operations and use of grouping symbols	c Use and apply order of operations rules as appropriate for mental, paper/pencil and calculator usage
d Recognize and use concepts related to multiples, factors, primes and composites	d Recognize and use concepts related to multiples, factors, primes and composites; and determine divisibility by 2, 5, & 10	d Recognize and use concepts related to factoring and determining devisibility	d Recognize and use concepts related to factoring and determining devisibility	

2.0 Appropriate Computational Skills: Students select and use the most appropriate form of computation - manipulative, mental, paper/pencil, estimation or calculator usage to solve problems and check all computations for reasonability.

2.1 USE MENTAL, PAPER AND PENCIL, ESTIMATION AND CALCULATOR COMPUTATIONS TO SOLVE APPROPRIATE PROBLEMS (ELS 1.4 and 1.7)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Add and subtract with actual objects (up to 10)	a Add and subtract with actual objects (up to 10)	a Add and subtract actual objects (up to 20)	a Use mental, manual or calculator processes to perform grade-level arithmetic operations	a Use mental, manual or calculator processes to perform grade-level arithmetic operations
			b Select the most appropriate method of computation (manipulative, mental, paper/pencil, calculator) to use in a given situation	b Select the most appropriate method of computation (manipulative, mental, paper/pencil, calculator) to use in a given situation
	c Estimate and approximate answers to problems	c Use estimating skills, such as rounding, to make approximate whole number computations	c Use estimating skills, such as rounding, to make approximate whole number computations	c Use rounding and other techniques useful in mental computation to estimate and make approximate whole number, fraction and decimal computations
		d Apply acquired strategies including modeling patterns (such as "counting on," "doubles," "neighbors," etc.) and properties (commutativity and associativity), to aid in quick recall of addition, subtraction, and multiplication facts	d Apply acquired strategies including modeling patterns (such as "counting on," "doubles," "neighbors," etc.) and properties (commutativity and associativity), to aid in quick recall of addition, subtraction and multiplication facts	d Apply acquired strategies to aid in quick recall of all basic facts
	e Mentally add 10 to a single number	e Mentally add or subtract multiples of 10 to (from) a number	e Solve mentally, appropriate addition and subtraction problems involving place value understanding, e.g., add or subtract 10 or 100 to (from) any 3-digit number; add or subtract multiples of 10 or 100	e Use mental arithmetic skills to solve appropriate problems (multiples of 10 and 100, addition of fractions with like denominators, etc.)

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Use mental, manual or calculator processes to perform grade-level arithmetic operations	a Use mental, manual or calculator processes to perform grade-level arithmetic operations	a Use mental, manual, calculator and computer processes to perform mathematical operations	a Use mental, manual, calculator and computer processes to perform mathematical operations	a Use mental, manual, calculator and computer processes to perform mathematical operations
b Select the most appropriate method(s) of computation (manipulative, mental, paper/pencil, calculator) to use in a given situation	b Select the most appropriate method(s) of computation (manipulative, mental, paper/pencil, calculator) to use in a given situation	b Select the most appropriate method(s) of computation (mental, paper/pencil, calculator) to use in a given situation	b Select the most appropriate method(s) of computation (mental, paper/pencil, calculator) to use in a given situation	b Select the most appropriate method(s) of computation (mental, paper/pencil, calculator) to use in a given situation
c Use rounding and other techniques useful in mental computation to estimate and make approximate whole number, fraction and decimal computations	c Use rounding and other techniques useful in mental computation to estimate and make approximate whole number, fraction and decimal computations	c Use rounding and other techniques useful in mental computation to estimate and make approximate whole number, fraction, decimal, and percent computations	c Use rounding and other techniques useful in mental computation to estimate and make approximate whole number, fraction, decimal, and percent computations	c Use and apply estimation techniques
d Apply acquired strategies to aid in quick recall of all basic facts				
e Solve mentally, appropriate whole number, fraction and decimal problems, e.g., 10 x 64; 60 x 20; 14,000 - 7,000; 5,000 + 261; 3,000 x 7; 1/4 + 3/4; 5/8 - 4/8; 3 - 0.5	e Mentally +, -, x, ÷ whole numbers and decimals by powers of ten and multiples of ten, and compute mentally among commonly used fractions and decimals	e Mentally +, -, x, ÷ whole numbers and decimals by powers of ten and multiples of ten, percents such as 25% and 50%, and compute mentally among commonly used fractions and decimals	e Mentally +, -, x, ÷ whole numbers and decimals by powers of ten and multiples of ten and commonly used percents	e Compute mentally when appropriate (e.g., 7 x 20; $1.50 + $.35; 4 - 1 1/4; $1.25 + $5, $7.29 + $3.82; $7.09 + $3.82 is approximately $11 or 17 15/16" - 9 1/2" is approximately 8 1/2")

(2.1 cont.)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
		f Perform addition and subtraction algorithms with and without regrouping using 1-2 digit whole numbers	f Perform addition and subtraction algorithms with and without regrouping using 1-3 digit whole numbers	f Use paper/pencil to perform addition, subtraction, multiplication of whole numbers, 1-digit division, addition and subtraction of decimals, addition and subtraction of fractions with like denominators
				g Compute using measures of length, weight (mass), time and money
h Explore counting, addition and subtraction with a calculator	h Explore counting, addition and subtraction with a calculator	h Explore counting, addition and subtraction with a calculator	h Use a calculator to solve appropriate problems and to check approximate calculations (e.g., real problems with lengthy calculations or large numbers)	h Use calculator and/or computer to solve appropriate problems (e.g., real problems with lengthy calculations or large numbers)
	j Examine all answers for reasonableness	j Examine all answers for reasonableness	j Use estimation and other skills to check answers for reasonableness	j Examine all answers for reasonableness
k Recall informal experiences with multiplication and division	k Engage in informal experiences with multiplication and division	k Solve addition/subtraction word problems and create addition/subtraction word problems to match a number sentence		

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
f Use paper/pencil to perform addition, subtraction, multiplication of whole numbers, 1-digit division, addition and subtraction of decimals, addition and subtraction of fractions with like denominators	f Use paper/pencil to perform addition, subtraction, multiplication of whole numbers, 1-digit division, addition and subtraction of decimals, addition and subtraction of fractions	f Perform paper/pencil arithmetic computation on commonly used whole numbers, decimals and fractions	f Recognize that the same arithmetic algorithms used with 1-3 digit numbers can be extended to multi-digit computations	f Recognize that the same arithmetic algorithms used with 1-3 digit numbers can be extended to multi-digit computations
g Compute using measures of length, weight (mass), time and money	g Compute using measures of length, weight (mass), time and money	g Compute using measures	g Compute using measures	g Compute using measures
h Use calculator and/or computer to solve appropriate problems (e.g., real problems with lengthy calculations or large numbers)	h Use calculator and/or computer to solve appropriate problems	h Use calculator and/or computer to solve appropriate problems	h Use calculator and/or computer to solve appropriate problems and recognize the limitations of a calculator when dealing with large numbers	h Use calculator and/or computer to solve appropriate problems and recognize the limitations of a calculator when dealing with large numbers
			i Convert mentally, among decimals, percents and commonly used fractions	i Convert mentally, manually decimals, percents and commonly used fractions
j Use estimation and other skills to check answers for reasonableness	j Use estimation and other skills to check answers reasonableness	j Use estimation and other skills to check answers reasonableness	j Use estimation and other skills to check answers reasonableness	j Use estimation and other skills to check answers reasonableness

2.2 DEMONSTRATE COMPUTATIONAL ALGORITHMS WITH CONCRETE MATERIALS OR REAL-WORLD EXAMPLES

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
[a] Use concrete models to perform one digit addition/subtraction problems	[a] Use concrete models to perform one digit addition/subtraction problems	[a] Use concrete models to perform two digit addition/subtraction problems, and demonstrate place value exchanges (borrowing and carrying) up to 1000	[a] Use concrete models to perform whole number computations, and demonstrate place value exchanges (borrowing and carrying) up to 1000 and to model the various meanings of multiplication	[a] Use concrete materials to model various meanings of multiplication and division and to interpret remainders
				[c] Use models such as money or metrics to demonstrate addition and subtraction of decimals and multiplication of decimals by whole numbers
				[d] Use concrete models to demonstrate addition and subtraction of fractions with like denominators

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Use concrete materials to model the various meanings of multiplication and division and to interpret remainders	a Use concrete materials to model the various meanings of multiplication and division and to interpret remainders	a Demonstrate an understanding of the various meanings of multiplication and division (including remainders) of whole numbers by drawings or by referencing "real world" applications	a Demonstrate an understanding of the various meanings of multiplication and division (including remainders) of whole numbers by drawings or by referencing "real world" applications	
		b Model the various meanings of addition involving signed numbers in situations meaningful to students	b Model the various meanings of addition and subtraction involving signed numbers, and add, subtract, and multiply signed numbers in situations meaningful to students	b Use concrete or pictorial models to demonstrate addition, subtraction and multiplication of signed numbers
c Use models such as money or metrics to demonstrate addition, subtraction and multiplication of decimals by whole numbers	c Use models such as money or metrics to demonstrate addition, subtraction, multiplication and division of decimals by whole numbers	c Use models such as money or metrics to demonstrate addition, subtraction, multiplication and division of decimals	c Use concrete materials or "real world" examples to demonstrate operations with decimals and percents	c Use concrete materials or "real world" examples to demonstrate operations with decimals and percents
d Use concrete models to demonstrate addition and subtraction of commonly used fractions and to model the multiplication of a fraction by a whole number and by a fraction using the "of" concept	d Use rulers, grid paper, and/or other intuitive methods for +, −, and x with commonly used fractions	d Use rulers, grid paper, and/or other intuitive methods for +, −, x, and ÷ with commonly used fractions	d Use concrete materials or "real world" examples to demonstrate operations with commonly used fractions	d Use concrete materials or "real world" examples to demonstrate operations with commonly used fractions

3.0 Problem Solving: Students use problem-solving skills and strategies to solve routine and nonroutine problems.

3.1 IDENTIFY PROBLEMS AND APPROACH THEIR SOLUTION IN AN ORGANIZED MANNER (ELS 6.3)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Identify problems that need a solution	a Identify problems that need a solution	a Identify problems that need a solution	a Identify problems that need a solution	a Identify problems, recognize information necessary to solve problems, and supply additional information if needed
b Use simple questioning to clarify problems	b Use simple questioning to clarify problems	b Use simple questioning to clarify problems	b Use simple questioning to clarify problems	b Use simple questioning strategies to clarify problems
	c Use data from questioning to develop a problem-solving plan	c Use data from questioning to develop a problem-solving plan	c Use data from questioning to develop a problem-solving plan	c Use data from the questioning process to develop a problem-solving plan
d Solve problems using strategies such as guessing and checking, using concrete objects, making a model, generating a pattern or drawing a picture	d Solve problems using strategies such as guessing and checking, using concrete objects, making a model, generating a pattern or drawing a picture	d Solve problems using strategies such as guessing and checking, using concrete objects, making a model, generating a pattern or drawing a picture	d Solve problems using strategies such as guessing and checking, using concrete objects, making a model, generating a pattern or drawing a picture	d Solve problems using a variety of strategies such as guessing and checking, making predictions based upon a pattern, making a drawing or model
	e Identify alternative solutions to a simple problem	e Identify alternative solutions to a simple problem	e Identify alternative solutions to a simple problem	e Identify alternative solutions to problems
	f Choose and apply mental, manual and calculator processes to problem-solving strategy(ies)	f Choose and apply mental, manual and calculator processes to problem-solving strategy(ies)	f Choose and apply mental, manual and calculator processes to problem-solving strategy(ies)	f Choose and apply mental, manual and calculator processes to problem-solving strategy(ies)

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Identify problems, recognize information necessary to solve problems, and supply additional information if needed	a Define a problem, choose information to solve the problem and supply additional information, if needed	a Define a problem, choose information to solve the problem and supply additional information, if needed	a Define a problem, choose information to solve the problem and supply additional information, if needed	a Define a problem, choose information to solve the problem and supply additional information, if needed
b Use simple questioning strategies to clarify problems	b Use a combination of questioning strategies and observation to analyze problems	b Use a combination of questioning strategies and observation to analyze problems	b Use a combination of questioning strategies and observation to analyze problems	b Apply recognized research techniques to analyze problems
c Use data from the questioning process to develop a problem-solving plan	c Use data from several sources to develop a problem-solving plan	c Use data from several sources to develop a problem-solving plan	c Use data from several sources to develop a problem-solving plan	c Design and carry out a plan for solving a problem
d Solve problems using a variety of strategies such as guessing and checking, making predictions based upon a pattern, making a drawing or model	d Solve problems using problem-solving skills such as guessing and checking, looking for a pattern, making systematic lists, making a drawing or model, eliminating possible answers, or solving a simpler related problem	d Solve problems using problem-solving skills such as guessing and checking, looking for a pattern, making systematic lists, making a drawing or model, eliminating possible answers, or solving a simpler related problem	d Solve problems using appropriate strategies such as guessing and checking, making a systematic list, looking for patterns, making or drawing a model, eliminating possible answers or solving a simpler problem	d Solve problems using the most appropriate tools, methodologies, processes and operations in solving a variety of problems
e Identify alternative solutions to problems	e Identify alternative solutions to problems	e Identify alternative solutions to problems	e Identify alternative solutions to problems	e Identify alternative solutions to problems
f Choose and apply mental, manual and calculator processes to problem-solving strategy(ies)	f Choose and apply mental, manual, calculator, and computer processes to problem-solving strategy(ies)	f Choose and apply mental, manual, calculator, and computer processes to problem-solving strategy(ies)	f Choose and apply mental, manual, calculator, and computer processes to problem-solving strategy(ies)	f Choose and apply mental, manual, calculator, and computer processes to problem-solving strategy(ies)
	g Select and apply appropriate problem-solving tools, including computer software	g Select and apply appropriate problem-solving tools, including computer software	g Select and apply appropriate problem-solving tools, including computer software	g Select and apply appropriate problem-solving tools, including computer software

(3.1 cont.)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
h Share successful and unsuccessful problem-solving strategies	h Share successful and unsuccessful problem-solving strategies	h Share successful and unsuccessful problem-solving strategies	h Share successful and unsuccessful problem-solving strategies	h Share successful and unsuccessful problem-solving strategies
i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies
	j Develop new suggestions or approaches if problem is not solved	j Develop new suggestions or approaches if problem is not solved	j Develop new suggestions or approaches if problem is not solved	j Use formative (in process) data to modify or confirm problem-solving plan

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
h Share successful and unsuccessful problem-solving strategies	h Share successful and unsuccessful problem-solving strategies	h Describe both successful and unsuccessful solution strategies	h Describe both successful and unsuccessful solution strategies	h Evaluate problem-solving strategies in terms of tools, methodologies, processes, operations
i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies	i Engage in cooperative problem solving and compare alternative solution strategies
j Use formative (in process) data to modify or confirm problem-solving plan	j Use formative (in process) data to modify or confirm problem-solving plan	j Use summative (final) data to determine if the problem-solving approach was successful, and if not, how it should be modified	j Use summative (final) data to determine if the problem-solving approach was successful, and if not, how it should be modified	j Analyze formative and summative data to confirm or revise the proposed solution

3.2 CREATE AND SOLVE WORD PROBLEMS APPROPRIATE TO THE GRADE LEVEL (ELS 6.3)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Role play put together and take away situations	a Recognize appropriate operation(s) (+, -) for solutions of word problems	a Recognize appropriate operation(s) (+, -) for solutions of word problems	a Recognize appropriate operation(s) (+, -, x) for solutions of word problems	a Recognize appropriate operation(s) (+, -, x, -) for solutions of word problems
b Solve simple problems by role playing	b Solve simple word problems by role playing	b Solve one-step problems presented verbally by role playing and use of manipulation	b Solve one-step word problems including those involving money, measurement, and data presented in graphs, tables and charts	b Solve one-step word problems including those involving money, measurement, and data presented in graphs, tables and charts
	c Create addition/ subtraction word problems to match number sentences	c Create addition/ subtraction word problems to match number sentences	c Create word problems to match addition, subtraction and multiplication algorithms	c Create word problems to match addition, subtraction, multiplication, and division algorithms
	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Recognize appropriate operation(s) for solutions of word problems, and recognize information necessary to solve word problems, and supply reasonable additional information, if needed	a Recognize appropriate operation(s) for solutions of word problems, and recognize information necessary to solve word problems, and supply reasonable additional information, if needed	a Recognize appropriate operation(s) for solutions of word problems, and recognize information necessary to solve word problems, and supply reasonable additional information, if needed	a Recognize appropriate operation(s) for solutions of word problems, and recognize information necessary to solve word problems, and supply reasonable additional information, if needed	a Recognize appropriate operation(s) for solutions of word problems, and recognize information necessary to solve word problems, and supply reasonable additional information, if needed
Solve one- and two-step word problems including those involving money, measurement, and data presented in graphs, tables and charts	b Pose new problems and solve multiple step word problems	b Pose new problems and solve multiple step word problems	b Pose and solve multiple-step word problems	b Pose and solve word problems
c Create word problems to match whole number, fraction and decimal algorithms	c Create word problems to match whole number, fraction and decimal algorithms	c Solve and create word problems to match exercises involving whole numbers, fractions, decimals, and percent	c Solve and create word problems to match exercises involving whole numbers, fractions, decimals, and percent	c Solve and create problems to match exercises involving ratios, proportions and formulas
			d Identify, invent, or create problems that can be solved by using ratio and proportion; and use proportion to solve problems	d Identify, invent, or create problems that can be solved by using ratio and proportion; and use proportion to solve problems
e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information	e Solve problems with more than one possible solution and recognize problems which cannot be solved because they contain too little information
		f Translate "real-world" problems into mathematical statements, and mathematical problems and answers back into "real-world" context	f Translate "real-world" problems into mathematical statements, and mathematical problems and answers back into "real-world" context	f Translate "real-world" problems into mathematical statements, and mathematical problems and answers back into "real-world" context

4.0 Geometry and Visualization Skills: Students recognize geometric patterns and relationships and apply them in solving problems and making predictions.

4.1 RECOGNIZE AND USE GEOMETRIC PATTERNS, RELATIONSHIPS AND PRINCIPLES TO DESCRIBE AND CLASSIFY (ELS 1.5)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Identify the attributes of rectangle, circle and triangle	a Identify the attributes of rectangle, circle and triangle	a Identify similar and different attributes of two or more geometric figures	a Identify similar and different attributes of two or more geometric figures	a Identify attributes of polygons and common geometric solids
b Informally explore 2-D shapes	b Informally explore 2-D shapes	b Sketch, model and use 2-D shapes	b Identify, sketch, model and manipulate squares, rectangles, circles, triangles, cubes	b Sketch, model and manipulate squares, rectangles, circles, triangles, cubes, cones, and cylinders
c Informally explore symmetry (e.g., pattern blocks, in nature)	c Through informal experiences identify symmetry in the environment and create designs with symmetry (e.g., construction with colored tiles or cubes)	c Identify symmetry in environment, and create designs with symmetry (e.g., construction with colored tiles or cubes)	c Identify symmetry and geometric forms in the environment (e.g., construction with colored tiles or cubes)	c Complete constructions and designs which have one or more lines of symmetry and identify parallel and intersecting lines and right angles in the environment
d Copy or extend patterns using concrete models	d Copy or extend patterns using concrete models and drawing pictures	d Copy or extend patterns using concrete models and drawing pictures	d Copy or extend patterns using concrete models and drawing pictures	d Copy or extend patterns using concrete models and drawing pictures
				e Locate coordinate points on graph paper

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Identify properties of common geometric figures, including quadrilaterals and geometric solids	a Identify properties of common geometric figures, including quadrilaterals and geometric solids	a Identify distinguishing properties of common geometric figures, including side or angle measurements	a Identify distinguishing properties of common geometric figures, including side or angle measurements	a Identify and compare common two- and three-dimensional geometric shapes and solids according to attributes and properties
b Draw or model simple, common geometrical figures with specific dimensions using ruler, tangrams, squared paper, or other concrete materials	b Draw or model simple, common geometrical figures with specific dimensions using ruler, tangrams, squared paper, or other concrete materials	b Sketch or build common geometric solids and two-dimensional figures	b Sketch or build common geometric solids and two-dimensional figures	b Model or make drawings of 2- or 3-dimensional shapes and solids useful in solving problems
c Identify, sketch or model intersecting lines, right angles and lines of symmetry occurring in the environment	c Demonstrate parallel, intersecting, or skew lines, measure angles and identify lines of symmetry	c Identify, sketch or model parallel and intersecting lines, right angles and lines of symmetry occurring in the environment	c Identify, sketch or model parallel and intersecting lines, right angles and lines of symmetry occurring in the environment	c Recognize and apply the concepts of symmetry, congruency and similarity of geometrical figures as commonly used in man-made objects
d Copy or extend patterns using concrete models and drawing pictures	d Use drawings, models or computers to demonstrate geometric patterns and relationships such as congruence	d Use drawings, models or computers to demonstrate geometric patterns and relationships such as similarity and congruence	d Use drawings, models or computers to demonstrate geometric patterns and relationships such as similarity and congruence	d Use drawings, models or computers to demonstrate geometric patterns and relationships such as similarity and congruence
e Locate points on graph paper, maps and globes, and graph coordinates (emphasize examples from the environment)	e Locate points on graph paper, maps and globes, and graph coordinates (emphasize examples from the environment)	e Locate and give coordinates of points on graph paper, maps, globes (emphasize examples from the environment and computer applications)	e Locate and give coordinates of points on graph paper, maps, globes and other charts (emphasize examples from the environment and computer applications)	e Locate points and lines and determine distance and area in a rectangular coordinate system (emphasize examples from the environment and computer applications)
f Recognize the concepts of diameter, radius and circumference	f Recognize the concepts of diameter, radius and circumference	f Demonstrate by various means the relationships among radius, diameter and circumference of a circle	f Demonstrate by various means the relationships among radius, diameter and circumference of a circle, and the common right triangle relationships	f Apply and use circle and common right triangle relationships in solving problems

4.2 MAKE AND USE GEOMETRIC DRAWINGS AND MODELS, INCLUDING TESSELLATIONS

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
[a] Explore with manipulatives (e.g., pattern blocks, or tiles)	[a] Explore with manipulatives (e.g., pattern blocks, or tiles)	[a] Explore with manipulatives (e.g., pattern blocks, or tiles)	[a] Create simple constructions with tiles and copy on squared paper	[a] Investigate covering flat surfaces with various congruent polygons
				[b] Draw rectangular prisms from a side view
				[c] Use cubes to build structures suggested by pictures, and materials such as pattern blocks, and grid paper to investigate certain number patterns (e.g., square numbers, primes, area model of multiplication facts)

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Find and use congruent polygons which will cover a surface without overlapping	a Show how a flat surface can be completely covered, without overlap, by congruent rectangles, triangles or squares	a Show how a flat surface can be completely covered, without overlap, by congruent rectangles, triangles or squares	a Explain why a flat surface can be completely covered without overlap, by congruent triangles, and rectangles, or squares	
b Sketch top and side views of rectangular solids	b Make 2-dimensional sketches of geometric solids such as pyramids, cones, spheres, cubes	b Make 2-dimensional sketches of geometric solids such as pyramids, cones, spheres, cubes	b Use protractor, compass, ruler, computer, and other instruments to make common geometric constructions	b Use protractor, compass, ruler, computer, and other instruments to make common geometric constructions
c Use cubes to build structures suggested by pictures and make 3-D shapes from paper patterns	c Construct a stack of cubes given pictures of front, top and side views and make 3-D shapes from paper patterns	c Construct a stack of cubes given pictures of front, top and side views	c Draw the net (2 dimentional pattern) for common geometric solids, e.g., cube, rectangular prism, cylinder	c Draw the net (2 dimentional pattern) for common geometric solids, make orthographic and isometric drawings of structures built with cubes, and visualize and draw cross-sections formed by slicing geometric solids
		d Show the importance of using triangles in construction	d Show the importance of using triangles in construction and surveying by using a drawing or model	d Use and apply triangles in construction and surveying

4.3 UNDERSTAND AND USE PERIMETER, AREA AND VOLUME CONCEPTS

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Explore with manipulatives to gain experiences with measurement of perimeter, area and capacity	a Explore with manipulatives to gain experiences with measurement of perimeter, area and capacity	a Explore with manipulatives to gain experiences with measurement of perimeter, area and capacity	a Develop an understanding of perimeter, area and volume using concrete objects	a Develop an understanding of perimeter, area and volume using concrete objects
b Estimate and determine "space covered" using square tiles or other uniform objects	b Estimate and determine "space covered" using square tiles or other uniform objects	b Estimate and determine "space covered" using square tiles or other uniform objects	b Use common objects to estimate perimeter, area and volume	b Use common objects to estimate perimeter, area and volume

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Develop an understanding of perimeter, area and volume using concrete objects	a Develop an understanding of perimeter, surface area and volume using concrete objects	a Demonstrate, other than by using a formula, ways of finding perimeter and area of right triangles, circles, rectangles	a Demonstrate, other than by using a formula, ways of finding perimeter and area of general triangles, circles, parallelograms, and trapezoids	a Use and apply perimeter, area and volume concepts
b Estimate and determine perimeter and area of rectangles, and volume of rectangular solids, by means other than formula	b Estimate and determine perimeter and area of rectangles and right triangles, and volume of rectangular solids, by means other than formula	b Estimate and determine area of right triangles, circles, rectangles, and volume of rectangular solids	b Estimate and determine perimeter, area and volume of common geometric figures	b Estimate perimeter, area and volume
c Use formula for finding perimeter and area of rectangles	c Use formula for finding perimeter and area of rectangles, and volume of rectangular solids	c Use a formula for finding perimeter and area of right triangles, circles, rectangles, and volume of rectangular solids	c Use a formula for finding perimeter, area and volume of common geometric figures	c Calculate perimeter, area and volume
		d Calculate the surface areas of regularly shaped solids (e.g., rectangular boxes and cubes)	d Calculate surface areas of regularly shaped solids (e.g., cubes, cylinders, rectangular boxes)	d Calculate surface areas of regularly shaped solids (e.g., cubes, cylinders, rectangular boxes)

5.0 Measurement: Students measure quantities and use measurements to keep records, solve problems and make predictions.

5.1 POSE AND SOLVE PROBLEMS THAT INVOLVE TIME AND MONEY (ELS 1.7)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Informal experiences with money	a Identify and order by value (penny, nickel and dime) and role play making change up to a dime	a Identify and order all U.S. coins, role play making change and show equivalent amounts up to a quarter	a Identify and order by value; make change using U.S. coins	a Identify and order by value, make change using U.S. currency, and use bills and coins to show equivalent amounts to $5
b Demonstrate an understanding of time (day, week, month, tomorrow, yesterday)	b Demonstrate an understanding of time (day, week, month, year, tomorrow, yesterday)	b Demonstrate an understanding of time (day, week, month, year, tomorrow, yesterday) and order days of the week	b Read time using standard and digital clocks and order months, seasons and days of week	b Read time using standard and digital clocks
		c Create and solve word problems which involve money	c Create and solve word problems which involve money or time	c Create and solve word problems which involve money or time

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Identify and order by value, and make change using U.S. currency				
b Estimate elapsed time for given activities				
c Create and solve word problems which involve money or time	c Create and solve problems which involve time or money	c Create and solve problems which involve time or money	c Create and solve problems which involve time or money including consumer and wage earner situations of interest to students	c Create and solve problems which involve time and money, including consumer and wage earner situations of interest to students

5.2 SELECT AND USE APPROPRIATE INSTRUMENTS AND UNITS TO ESTIMATE AND MEASURE LENGTH-WEIGHT; VOLUME AND CAPACITY; AND TEMPERATURE (ELS 1.7)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
[a] Provide numerous informal experiences, estimating and measuring length, weight, and capacity using nonstandard uniform units and line up measuring tool with item to be measured	[a] Provide numerous informal experiences, estimating and measuring length, weight, and capacity using nonstandard uniform units and line up measuring tool with item to be measured	[a] Estimate and measure objects using nonstandard uniform units, recognize the need for standard units of measurement and begin using meters, centimeters, feet, yards to measure length; use a balance scale to determine heavier or lighter	[a] Estimate and determine length and weight (mass) using nonstandard, metric or English U.S. Customary units of measure and select and use appropriate instrument and unit for a measurement task in nonstandard, metric, or English	[a] Recognize and use meters, centimeters, feet, yards, and inches to measure; and select the most appropriate instrument and unit for a measurement task
				[c] Use squared paper, transparent grids, or other materials to estimate area/ perimeter of irregular closed figures

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Recognize and use meters, centimeters, feet, yards, and inches to measure and select the most appropriate instrument and unit for a measurement task	a Determine the most appropriate unit and instrument for a measurement task	a Determine the most appropriate unit and instrument for a measurement task	a Determine the most appropriate unit and instrument for a measurement task	a Determine the most appropriate unit and instrument for a measurement task
	b Estimate and then directly measure distances, angles, and other quantities to the nearest and smallest unit on the measuring scale	b Estimate and then directly measure distances, angles, and other quantities to the nearest and smallest unit on the measuring scale	b Estimate and directly measure distances, angles and other quantities, and indicate in some way the precision of the measurement, using metric and English (U.S. Customary)	b Estimate and directly measure length, area, volume, time, weight (mass) etc., with reasonable accuracy and/or round a measurement to a given unit
c Use squared paper, transparent grids, or other material to estimate area/perimeter of irregular closed figures	c Use squared paper, transparent grids, or other material to estimate area/perimeter of irregular closed figures	c Measure by some direct means the area of a polygon or some 2-D region with curves as boundaries	c Measure by some direct means the area of a polygon or some 2-D region with curves as boundaries, and the volume of 3-D objects	
	d Convert among units of length and weight (mass) within the same measurement system (English and metric)	d Convert among commonly used units of measurement within the same system		
			e Give examples of the importance of congruence and precision in society	e Determine the precision of a given measuring tool, (e.g., 1° for a standard protractor)
		f Explain why all measurements are approximations	f Explain why all measurements are approximations and why results of all computations with measurements are approximations	f Explain why all measurements are approximations and why results of all computations with measurements are approximations

(5.2 cont.)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
g Explore the concepts of weight (mass) using a balance scale and common classroom objects	g Explore the concepts of weight (mass) using a balance scale and common classroom objects	g Explore the concepts of weight (mass) using a balance scale and common classroom objects	g Explore the concepts of weight (mass) using a balance scale and common classroom objects	g Recognize and use grams, kilograms, ounces, and pounds to measure common classroom objects
h Explore temperature concepts as they occur in daily situations	h Explore temperature concepts as they occur in daily situations	h Explore temperature concepts as they occur in daily situations	h Estimate, read and record temperature in C° and F°	h Estimate, read and record temperature in C° and F°

5.3 DETERMINE INDIRECT MEASUREMENTS (ELS 1.7)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
				a Estimate distances between places represented by points on maps

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GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
g Estimate and determine the weight (mass) of common classroom objects in metric and English (U.S. Customary) units	g Estimate and determine the weight (mass) of common classroom objects in metric and English (U.S. Customary) units	g Estimate and determine the weight (mass) of common classroom objects in metric and English (U.S. Customary) units	g Estimate and determine the weight (mass) of common objects using metric and English (U.S. customary) units	
h Estimate, read and record temperature in C° and F°	h Estimate, read and record temperature in C° and F°	h Estimate, read and record temperature in C° and F°	h Estimate, read and record temperature in C° and F° in real and given situations	

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Locate points, give coordinates of points on maps, and estimate distances between places represented by points on maps	a Locate points and give coordinates of points on maps, make scale drawings and determine highway distances from maps	a Make scale drawings and determine actual distances from scale drawings, blueprints, maps, and globes	a Make scale drawings and determine actual distances from scale drawings, blueprints, maps, and globes	a Apply ratio and proportion concepts in making and using scale drawings and models, and in solving problems
	b Find measurements, including inaccessible distances, indirectly by using formulas for distance and area	b Find measurements, including inaccessible distances, indirectly by using formulas for distance and area	b Find and record measurements using proportions and formulas	b Find and record measurements using proportions and formulas

6.0 Statistics and Probability: Students will be able to collect, organize, record and interpret data and be able to predict probable outcomes based on collected data.

6.1 RECOGNIZE AND USE MATHEMATICAL PATTERNS, RELATIONSHIPS AND PRINCIPLES TO QUANTIFY PROBLEMS OR MAKE PREDICTIONS (ELS 1.6)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Experience activities which have outcomes that depend on chance	a Experience activities which have outcomes that depend on chance	a Experience activities which have outcomes that depend on chance	a Apply intuitive probability concepts (e.g., make predictions in games by using terms such as "more likely," "less likely," "fair," etc.)	a Collect data from probability experiments
		b Make predictions using terms such as "best chance," "most likely," "least likely," "fair," etc.; apply intuitive probability concepts in games and activities (dice, spinners, etc.)	b Make predictions using terms such as "best chance," "most likely," "least likely," "fair," etc.; apply intuitive probability concepts in games and activities (dice, spinners, etc.)	b Recognize the concept of fair or unfair in game situations
d Collect and record data (e.g., tally, lists, charts)	d Collect and record data (e.g., tally, lists, charts)	d Collect and record data (e.g., tally, lists, charts)	d Collect and record data (e.g., tally, lists, charts)	d Gather and record data from a variety of sources
e Make and draw conclusions from real graphs (an array of actual objects on rectangular grid) picture graphs, bar graphs and charts	e Make and draw conclusions from real graphs, picture graphs, bar graphs and charts	e Make and draw conclusions from picture graphs, bar graphs, and charts	e Collect and record data from picture graphs, bar graphs and charts to draw conclusions and make predictions	e Collect and record data from picture graphs, bar graphs and charts to draw conclusions and make predictions
f Make and use real graphs (i.e., an array of actual objects on rectangular grid)	f Make and use real graphs and simple picture graphs	f Make and use real graphs and simple picture graphs	f Make and use picture graphs and bar graphs	f Make bar graphs, and organize information into tables/charts and diagrams given appropriate scale (e.g., box plots, box-and-whiskers, line plots, Venn diagrams)
				g Collect random samples

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Generate, record and interpret data from probability experiments and predict chances of an outcome	a Determine the number of possible events that could occur in a probability experiment (spinners, dice)	a Determine the number of possible events that could occur in a probability experiment (spinners, dice)	a Determine the number of possible events and the probability of an outcome in a probability experiment	a Interpret everyday uses of probability such as weather predictions, election forecasts or chances of winning a lottery
b Recognize the concept of fair or unfair in game situations	b Identify and demonstrate situations in which probability or chance of an event occurring is likely, unlikely, equally likely; and whether a game is "fair"	b Identify and demonstrate situations in which probability or chance of an event occurring is likely, unlikely, equally likely; and whether a game is "fair"	b Identify and demonstrate situations in which probability or chance of an event occurring is likely, unlikely, equally likely; and whether a game is "fair"	b Identify and demonstrate situations in which probability or chance of an event occurring is likely, unlikely, equally likely; and whether a game is "fair"
c Recognize certain (1) and impossible (0) probabilities	c Recognize certain (1) and impossible (0) probabilities	c Understand the meaning of probabilities of 0 (impossible) and 1 (certain)	c Understand the meaning of probabilities of 0 (impossible) and 1 (certain)	
d Use charts, tables and lists to organize all possible outcomes of an experiment	d Use charts, tables and lists to organize all possible outcomes of an experiment	d Use charts, tables and lists to organize all possible outcomes of an experiment	d Use charts, tables and lists to organize all possible outcomes of an experiment	
e Read, interpret, construct bar graphs, line graphs, tables and charts and make predictions based upon them	e Read and interpret graphs, tables and charts, and make predictions based upon them	e Read and interpret graphs, tables and charts, and make predictions based upon them	e Read and interpret graphs, tables and charts, and make predictions based upon them	e Read and interpret graphs, tables and charts and make predictions based upon them
f Organize information into tables/charts and diagrams given appropriate scale, e.g., box plots, box-and-whiskers, line plots, Venn diagrams	f Make histograms and bar graphs from data meaningful to students	f Make histograms and bar and line graphs from data meaningful to students	f Make line and circle graphs from data meaningful to students	f Organize and display data using tables, charts, graphs, and diagrams
g Collect random samples	g Use data gathering procedures which will aid in answering questions of interest to students (conducting polls, using tables from almanacs)	g Use data gathering procedures which will aid in answering questions of interest (conducting polls, using tables from almanacs)	g Use data gathering procedures which will aid in answering questions of interest (conducting polls, sampling schemes)	g Use data gathering procedures which will aid in answering questions of interest (conducting polls, sampling schemes)

(6.1 cont.)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
				j Read and interpret computer generated graphs and tables
l Predict simple possible future outcomes or actions	l Predict simple possible future outcomes or actions	l Predict simple possible future outcomes or actions	l Predict simple possible future outcomes or actions	l Predict simple possible future outcomes or actions

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
		h Understand the relationship between size of sample and degree of certainty	h Understand the relationship between size of sample and degree of certainty	h Understand the relationship between size of sample and degree of certainty
	i Identify misleading or incorrect methods of displaying or interpreting data	i Identify misleading or incorrect methods of displaying or interpreting data	i Identify misleading or incorrect methods of displaying or interpreting data	i Identify misleading or incorrect methods of displaying or interpreting data
Read and interpret computer generated graphs and tables	j Read and interpret computer generated graphs and tables	j Use computer software to generate graphs and tables	j Use computer software to generate graphs and tables	j Use computer software to generate graphs and tables
			k Show the relationship among variables using tables, graphs, formulas, and models	k Show the relationship among variables using tables, graphs, formulas, and models
l Predict simple, possible future outcomes or actions	l Predict simple, possible future outcomes or actions	l Predict probable future outcomes or actions	l Predict probable future outcomes or actions	l Defend conclusions from information given
	m Determine mean, median, and mode from data meaningful to students	m Determine mean, median, and mode from data meaningful to students	m Determine, interpret and compare advantages and disadvantages of mean, median and mode	m Collect, display, interpret statistical data using mean, mode, median, range and percentile

6.2 GENERATE AND TEST INTERPRETATIONS, EXPLANATIONS, PREDICTIONS, AND HYPOTHESES (ELS 6.2)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
			a Identify facts that support an explanation and a prediction	a Identify facts that support an explanation and a prediction
			b Identify factors that may influence a behavior or a result	b Identify factors that may influence a behavior or a result
			c Identify ways to determine whether a duplicate of an experiment will produce the same results	c Identify ways to determine whether a duplicate of an experiment will produce the same results

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Identify parts of an explanation and a prediction not supported by fact	a Identify parts of an explanation and a prediction not supported by fact	a Interpret differences between two explanations	a Interpret differences between two explanations	a Critically analyze explanations and interpretations to confirm or validate them
b Predict what influence different factors will have on a behavior or result	b Predict what influence different factors will have on a behavior or result	b Develop a hypothesis from observed data	b Develop a hypothesis from observed data	b Develop a hypothesis using data from a variety of sources
c Follow directions to conduct an experiment and identify the hypothesis used	c Follow directions to conduct an experiment and identify the hypothesis used	c Gather data that confirms or negates a hypothesis	c Gather data that confirms or negates a hypothesis	c Design and conduct a test of a hypotheses and report the results

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7.0 Mathematical Relationships: Students recognize and use number patterns, relationships, and logical thinking skills to make predictions and to solve problems.

7.1 SORT AND CLASSIFY; USE LOGICAL THINKING (ELS 5.2)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Classify and sort objects using one or more attributes by observing relationships and by making generalizations	a Classify and sort objects using one or more attributes by observing relationships and by making generalizations	a Classify and sort objects using one or more attributes by observing relationships and by making generalizations	a Sort and classify objects by attributes	a Classify objects and simple geometrical figures by attributes
b Use evidence such as illustrations, examples, and verifiable sources to problem solve	b Use evidence such as illustrations, examples, and verifiable sources to problem solve	b Use evidence such as illustrations, examples, and verifiable sources to problem solve	b Use evidence such as illustrations, examples, and verifiable sources to problem solve	b Use evidence such as illustrations, examples, and verifiable sources to problem solve
c Select and organize details to problem solve	c Select and organize details to problem solve	c Select and organize details to problem solve	c Select and organize details to problem solve	c Select and organize details to problem solve
d Organize information or data using tallys and real graphs	d Organize information or data using tallys, tables, charts, and real graphs	d Organize information or data using tallys, tables, charts, and real graphs	d Organize information or data using formats such as outlining, making maps, tables, charts, and graphs	d Organize information or data using formats such as outlining, making maps, tables, charts, and graphs

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Classify objects and simple geometrical figures by attributes	a Classify geometrical figures and sets of numbers	a Classify geometrical figures and sets of numbers	a Classify geometrical figures and sets of numbers	a Classify geometrical figures and sets of numbers
b Use evidence such as illustrations, examples, and verifiable sources to problem solve	b Use evidence such as illustrations, examples, and verifiable sources to problem solve	b Use evidence from verifiable sources to support own ideas and concepts in problem solving	b Use evidence from verifiable sources to support own ideas and concepts in problem solving	b Use evidence from verifiable sources to support own ideas and concepts in problem solving
c Select and organize details to problem solve	c Select and organize details to problem solve	c Select and use details, examples, illustrations, evidence and logic to problem solve	c Select and use details, examples, illustrations, evidence and logic to problem solve	c Select and use details, examples, illustrations, evidence and logic to problem solve
d Organize information or data using formats such as outlining, making maps, tables, charts, and graphs	d Organize information or data using formats such as outlining, making maps, tables, charts, and graphs	d Organize information or data using formats such as outlining, making maps, tables, charts, graphs; and computer spread sheets	d Organize information or data using formats such as outlining, making maps, tables, charts, graphs; and computer spread sheets	d Organize information or data using formats such as outlining, making maps, tables, charts, graphs; and computer spread sheets

7.2 COMPREHEND MEANINGS OF WRITTEN, ORAL AND VISUAL COMMUNICATIONS INVOLVING NUMBER PATTERNS AND RELATIONSHIPS (ELS 3.1)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a State relationships using terms such as "greater than," "less than," and "equal to"	a State relationships using terms such as "greater than," "less than," and "equal to"	a State relationships using terms such as "greater than," "less than," and "equal to"	a State relationships using terms such as "greater than," "less than," and "equal to"	a State relationships using terms such as "greater than," "less than," and "equal to" and use the symbols >, <, =
b Relate new information to previous knowledge	b Relate new information to previous knowledge	b Relate new information to previous knowledge	b Relate new information to previous knowledge	b Relate new information to previous knowledge
c Draw logical conclusions from information presented	c Draw logical conclusions from information presented	c Draw logical conclusions from information presented	c Draw logical conclusions from information presented	c Draw logical conclusions from information presented
d Identify, verbalize, extend, and reproduce a pattern in a sequence of objects and numbers, and use to make predictions	d Identify, verbalize, extend, and reproduce a pattern in a sequence of objects and numbers, and use to make predictions	d Identify patterns in a sequence of objects or numerical patterns in charts and tables, and use to make predictions	d Find numerical patterns in charts and tables (e.g., 100-chart, addition and multiplication tables), and use number patterns and relationships to make predictions	d Find number patterns in 100-chart, addition, and multiplication tables (e.g., primes, odd and even, square numbers), and use number patterns and relationships to make predictions

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a State relationships using terms such as "greater than," "less than," and "equal to" and use the symbols >, <, =	a Use equality and inequality concepts and symbols	a Use equality and inequality concepts and symbols	a Use equality and inequality concepts and symbols	a Use equality and inequality concepts and symbols
b Relate new information to previous knowledge	b Relate new information to previous knowledge	b Relate new information to previous knowledge	b Relate new information to previous knowledge	b Relate new information to previous knowledge
c Draw logical conclusions from information presented	c Draw logical conclusions from information presented	c Draw logical conclusions from information presented	c Draw logical conclusions from information presented	c Synthesize information and draw conclusions
d Find numerical patterns in 100-charts and addition and multiplication tables (e.g., odd/even, primes, square numbers), and use patterns to complete simple charts and tables and to make predictions	d Find numerical patterns in 100-charts and addition and multiplication tables (e.g., odd/even, primes, square numbers), and use patterns to complete simple charts and tables and to make predictions	d Find numerical patterns and use to complete charts and tables	d Find numerical patterns and use to complete charts and tables	d Find numerical patterns and use to complete charts and tables
e Explore relationships found in tables of value	e Recognize relationships found in tables of value		e Recognize direct and indirect cause and effect relationships	e Infer direct and indirect cause and effect relationships
f Use decimal and fraction understandings and 100-grids to illustrate the "for every hundred" model of percent	f Use decimal and fraction understandings and 100-grids to illustrate the percent concept	f Show, using models such as 100-grids, number lines or a meter stick, how percent can be expressed as a fraction or decimal	f Show, using models such as 100-grids, number lines or a meter stick, how percent can be expressed as a fraction or decimal, and conversely	f Demonstrate fraction, decimal, percent relationships
			g Interpret and use the concepts of ratio, percent, proportion, and commonly occurring rates such as growth, speed and sports applications	g Interpret and use the concepts of ratio, percent, proportion, and commonly occurring rates such as growth, speed, interest and cost per unit

7.3 RECOGNIZE, CONSTRUCT AND DRAW INFERENCES CONCERNING RELATIONSHIPS AMONG THINGS AND IDEAS (ELS 6.1)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Identify characteristics of simple objects that remain the same even though some change occurs, e.g., cutting objects into two pieces	a Identify characteristics of simple objects that remain the same even though some change occurs, e.g., cutting objects into two pieces	a Identify characteristics of simple objects that remain the same even though some change occurs, e.g., cutting objects into two pieces	a Identify characteristics of simple objects that remain the same even though some change occurs, e.g., cutting objects into two pieces	
	b Make a simple table of values given a specific rule and match a table of values to its rule	b Make a simple table of values given a specific rule and match a table of values to its rule	b Make a simple table of values given a specific rule and match a table of values to its rule	b Make a simple table of values given a specific rule and match a table of values to its rule

7.4 REFLECT UPON AND IMPROVE OWN REASONING (ELS 6.6)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Describe in simple terms how a solution was reached	a Describe in simple terms how a solution was reached	a Describe in simple terms how a solution was reached	a Describe in simple terms how a solution was reached	a Describe the reasoning process being used
b Act upon suggestions for improving reasoning capabilities	b Act upon suggestions for improving reasoning capabilities	b Act upon suggestions for improving reasoning capabilities	b Act upon suggestions for improving reasoning capabilities	b Act upon suggestions for improving reasoning capabilities

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
b Make a simple table of values given a specific rule and match a table of values to its rule	b Make a simple table of values given a specific rule and match a graph or table of values to its formula	b Make a simple table of values given a specific rule and match a graph or table of values to its formula	b Evaluate or make a table for two-variable formulas which have meaning to students and match a graph or table of values to its formula	b Evaluate or make a table for two-variable formulas and match a graph or table of values to its formula
c Explore relationships illustrated in a table of values	c Describe the nature of change of each variable as suggested by a table of values	c Describe the nature of change of each variable as suggested by a table of values or graphs	c Describe the nature of change of each variable as suggested by a table of values, graph, or formula	c Describe the nature of change of each variable as suggested by a table of values, graph, or formula

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
Describe the reasoning process being used	a Describe the reasoning process being used	a Describe the reasoning process being used	a Describe the strengths and weaknesses of inductive and deductive reasoning	a Present arguments supporting the use of deductive or inductive reasoning for a particular purpose
b Act upon suggestions for improving reasoning capabilities				

8.0 Oral and Written Communication Skills: The student uses vocabulary, speech, numerals and other symbol systems essential for effective individual and group problem solving and for effective oral and written communication of mathematical concepts, problem-solving processes and results.

8.1 RECOGNIZE AND USE MATHEMATICS VOCABULARY COMMONLY USED IN GRADE-LEVEL MATERIALS (ELS 1.1)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Recognize and identify names and/or sounds of letter symbols	a Use phonetic analysis skills	a Use phonetic analysis skills	a Use phonetic analysis skills	a Use phonetic analysis skills
b Use clues within the environment in order to gain information from print	b Use illustrations and words in a sentence to infer correct word(s)	b Use illustrations and words in a paragraph to infer correct word(s)	b Use context clues in a paragraph to infer correct word(s)	b Use context clues in a paragraph to infer correct word(s)
c Make oral distinctions between compound and plural words	c Make oral distinctions between compound and plural words	c Recognize compound and plural words	c Distinguish compound and plural words	c Distinguish compound and plural words
d Recognize mathematical words that are common to the child's individual environment	d Recognize mathematical words that are common to the child's individual environment	d Use basic mathematical terms (such as sum, difference, less than, equal, greater than, equal, greater than, circle, triangle) to convey concepts of quantity, order, operation and shape	d Use basic mathematical terms (such as sum, total, difference, product, less than, equal, greater than, rectangle) to convey concepts of quantity, order, operation and shape	d Use basic mathematical terms (such as sum, total, difference, product, quotient, less than, equal, greater than, square) to convey concepts of quantity, order, operation and shape

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
b Use context clues in a passage to infer correct word(s)	b Use context clues in a passage to infer correct word(s)	b Use context clues in a selection to infer correct word(s)	b Use context clues in a selection to infer correct word(s)	b Use context clues in a selection to infer correct word(s)
Distinguish affixes and root words	c Recognize affixes and root words	c Distinguish affixes and root words	c Distinguish affixes and root words	c Distinguish affixes and root words
d Use mathematical terms to convey concepts of quantity, order, operation and shape (e.g., product, factor, quotient, remainder, sum, quadrilateral)	d Use basic mathematical terms and symbols to convey concepts of quantity, order, operation, and shape	d Use basic mathematical terms and symbols to convey concepts of quantity, order and operation	d Use basic mathematical terms and symbols to convey concepts of quantity, order, operation, and shape	D Use basic mathematical terms and symbols to convey concepts of quantity, order, operation, and shape
	e Recognize common abbreviations (e.g., ft., in.) and symbols (e.g., cm, kg)	e Recognize common abbreviations (e.g., ft., in.) and symbols (e.g., cm, kg)	e Recognize common abbreviations (e.g., ft., in.) and symbols (e.g., cm, kg)	e Recognize common abbreviations (e.g., ft., in.) and symbols (e.g., cm, kg)

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8.2 DETERMINE MEANING OF UNKNOWN WORDS COMMONLY USED IN MATHEMATICAL MATERIALS (ELS 1.2)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Use context clues from oral presentation to infer meaning of unknown word(s)	a Use illustrations and adjacent words in a paragraph to infer meaning of unknown word(s)	a Use illustrations and adjacent words in a paragraph to infer meaning of unknown word(s)	a Use adjacent words to infer meaning of unknown words	a Use adjacent words and/or context clues in a passage to infer meaning of unknown words
c Make oral distinctions between compound and plural words	c Make oral distinctions between compound and plural words	c Use knowledge of each part of a compound word to determine meaning	c Use knowledge of each part of a compound word to determine meaning	c Use knowledge of affixes and root words to determine word meanings
	d Use glossary in curriculum materials	d Use primary grade dictionaries	d Use dictionaries and glossaries in grade-level curriculum materials	d Use dictionary, glossary, and context to determine correct meaning of word

8.3 SPEAK WITH STANDARD PRONUNCIATION, APPROPRIATE VOLUME, RATE, GESTURES AND INFLECTIONS (ELS 1.3)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Listen and practice basic speech sounds	a Listen and practice basic speech sounds	a Listen and practice basic speech sounds	a Produce correct basic speech sounds	a Produce correct basic speech sounds
b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Use context clues to infer meaning of unknown words	a Use context clues, punctuation and syntax to infer meaning of unknown words and concepts	a Use context clues, punctuation and syntax to infer meaning of unknown words and concepts	a Use context clues, punctuation and syntax to infer meaning of unknown words and concepts	a Use context clues, punctuation and syntax to infer meaning of unknown words and concepts
b Recognize double meanings of words	b Recognize double meanings of words	b Recognize double meanings of words	b Recognize double meanings of words	b Recognize double meanings of words
c Use knowledge of affixes and root words to determine word meanings	c Use knowledge of affixes and root words to determine word meanings	c Use knowledge of affixes and root words to determine word meanings	c Use knowledge of affixes and root words to determine word meanings	c Use knowledge of affixes and root words to determine word meanings
d Use context to determine correct dictionary definition of word	d Use dictionaries, glossaries and other reference materials to find word meanings	d Use dictionaries, glossaries and other reference materials to find word meanings	d Use dictionaries, glossaries and other reference materials to find word meanings	d Use dictionaries, glossaries and definitions in footnotes to find word meanings

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Produce correct basic speech sounds	a Produce correct basic speech sounds			
b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English	b Pronounce words according to acceptable Standard English
			c Make oral presentations that use verbal and non-verbal communication skills effectively	c Make oral presentations that use verbal and non-verbal communication skills effectively

8.4 USE ORAL COMMUNICATION TO GIVE OR RECEIVE INFORMATION AND DIRECTIONS (ELS 2.3)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Paraphrase oral messages	a Paraphrase oral messages	a Paraphrase oral messages	a Paraphrase oral messages	a Paraphrase oral messages
b Give accurate oral directions	b Give accurate oral directions	b Give accurate oral directions	b Give accurate oral directions	b Give accurate oral directions
c Ask questions designed to clarify, gain assistance or locate information	c Ask questions designed to clarify, gain assistance or locate information	c Ask questions designed to clarify, gain assistance or locate information	c Ask questions designed to clarify, gain assistance or locate information	c Ask questions designed to clarify, gain assistance or locate information
d Share ideas and information orally with others	d Share ideas and information orally with others	d Share ideas and information orally with others	d Share ideas and information orally with others	d Share ideas and information orally with others
e Provide accurate descriptive detail orally	e Provide accurate descriptive detail orally	e Provide accurate descriptive detail orally	e Provide accurate descriptive detail orally	e Provide accurate descriptive detail orally
				f Take notes based on oral presentations and group discussions
g Follow 2-step oral instructions	g Follow 2-step oral instructions	g Follow 2-step oral instructions	g Follow 2-step oral instructions	g Follow 2-3-step oral instructions

8.5 DETERMINE THE SIGNIFICANCE AND ACCURACY OF INFORMATION AND IDEAS PRESENTED IN WRITTEN, ORAL, AURAL AND VISUAL COMMUNICATIONS (ELS 4.1)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Paraphrase oral messages	a Paraphrase oral messages	a Paraphrase oral messages	a Paraphrase oral messages	a Paraphrase oral messages
b Give accurate oral directions	b Give accurate oral directions	b Give accurate oral directions	b Give accurate oral directions	b Give accurate oral directions
c Ask questions designed to clarify, gain assistance or local information	c Ask questions designed to clarify, gain assistance or local information	c Ask questions designed to clarify, gain assistance or local information	c Ask questions designed to clarify, gain assistance or local information	c Ask questions designed to clarify, gain assistance or local information
d Share ideas and information orally with others	d Share ideas and information orally with others	d Share ideas and information orally with others	d Share ideas and information orally with others	d Share ideas and information orally with others
e Provide accurate descriptive detail orally	e Provide accurate descriptive detail orally	e Provide accurate descriptive detail orally	e Provide accurate descriptive detail orally	e Develop accurate detail based on oral explanations by others
f Take notes based on oral presentations and group discussions	f Take notes based on oral presentations and group discussions	f Take notes based on oral presentations and group discussions	f Take notes and prepare summaries based on oral presentations and group discussions	f Take notes and prepare summaries based on oral presentations and group discussions
g Follow 3-step oral instructions	g Follow 3-step oral instructions	g Follow multistep oral instructions	g Follow multistep oral instructions	g Follow multistep oral instructions

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
		a Separate between relevant and irrelevant information used to draw conclusions	a Separate between relevant and irrelevant information used to draw conclusions	a Distinguish between logical and illogical conclusions
		b Identify propaganda and other persuasion techniques (e.g., use and misuse of statistics)	b Identify propaganda and other persuasion techniques (e.g., use and misuse of statistics)	b Identify propaganda and other persuasion techniques (e.g., use and misuse of statistics)

8.6 USE ORAL COMMUNICATION TO INFLUENCE OTHERS AND TO RESPOND TO PERSUASION (ELS 4.2)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
			[a] Ask questions and draw reasonable conclusions from answers	[a] Ask questions and draw reasonable conclusions from answers

8.7 LISTEN, READ, VIEW AND EVALUATE PRESENTATIONS OF MASS MEDIA (ELS 4.4)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Provide logical answers based upon factual data	a Provide logical answers based upon factual data	a Provide logical answers based upon factual data	a Provide logical answers based upon factual data	a Provide logical answers based upon factual data
b Use multiple sources to verify information	b Use multiple sources to verify information	b Use multiple sources to verify information	b Use multiple sources to verify information	b Use primary and secondary source materials to verify information
				c Argue opposite sides of issues
				d Recognize sources of persuasion and select appropriate persuasive response
				e Use verbal persuasion techniques in a class presentation

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
		a Recognize persuasion techniques found in visual communications (e.g., use and misuse of graphs)	a Recognize persuasion techniques found in visual communications (e.g., use and misuse of graphs)	a Recognize persuasion techniques found in visual communications (e.g., use and misuse of graphs)

8.8 SELECT APPROPRIATE FORM OF WRITING (ELS 5.3)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
				[a] Write in a variety of forms such as reports, descriptions, or in problem posing or solving [b] Use writing appropriate to purpose such as to inform, pose problems or solve problems

8.9 PRESENT IDEAS IN UNDERSTANDABLE SEQUENCE ON THE TOPIC SELECTED (ELS 5.4)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4

8.10 SELECT AND USE GUAGE, GESTURES AND SYMBOLS APPROPRIATE TO PURPOSE, TOPIC AND SETTING WHEN MAKING ORAL PRESENTATIONS (ELS 5.5)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
			[a] Select words which make the meaning clear	[a] Select words which make the meaning clear [b] Plan and make oral and visual presentations

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Write in a variety of forms such as reports, descriptions, or in problem posing or solving	a Write in a variety of forms such as personal essays, journals, reports, descriptions, or in problem posing or solving	a Write in a variety of forms such as personal essays, journals, reports, descriptions, or in problem posing or solving	a Write in a variety of forms such as personal essays, journals, reports, descriptions, or in problem posing or solving	a Write in a variety of forms such as personal essays, journals, reports, descriptions, or in problem posing or solving
b Use writing appropriate to purpose such as to inform, pose problems or solve problems	b Use writing appropriate to purpose such as to inform, pose problems or solve problems	b Use writing appropriate to purpose such as to inform, pose problems or solve problems	b Use writing appropriate to purpose such as to inform, pose problems or solve problems	b Use writing appropriate to purpose such as to inform, pose problems or solve problems

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Write complete sentences	a Write complete sentences	a Write complete sentences	a Write complete sentences	a Write complete sentences
	b Write multiparagraph personal journals, reports or problem solution strategies	b Write multiparagraph personal journals, reports or problem solution strategies	b Write multiparagraph personal journals, reports or problem solution strategies	b Write multiparagraph personal journals, reports or problem solution strategies

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Select words which make the meaning clear	a Use a variety of techniques and figurative expressions to convey meaning	a Use a variety of techniques and figurative expressions to convey meaning	a Use a variety of techniques and figurative expressions to convey meaning	a Employ verbal, symbolic, graphic and visual techniques to convey information
b Plan and make oral and visual presentations	b Plan and make oral and visual presentations	b Plan and make oral and visual presentations	b Plan and make oral and visual presentations	b Plan and make oral and visual presentations

8.11 EVALUATE AND REVISE OWN WRITING FOR MEANING, CLARITY, AND COMPREHENSIVENESS (REVISING AND REWRITING) (ELS 5.6)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
			[a] Revise own writing to enhance clarity and meaning	[a] Revise own writing to enhance clarity and meaning [b] Use descriptive terms to emphasize facts and quantities

8.12 APPLY THE CONVENTIONS OF WRITING TO PRODUCE EFFECTIVE COMMUNICATION (EDITING AND PROOFREADING) (ELS 5.7)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
			[a] Edit for capitalization, end punctuation, and complete sentences	[a] Edit for complete and correct sentences, punctuation and usage [b] Spell correctly [c] Produce legible final copy

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Revise own writing to enhance clarity and meaning	a Revise own writing to enhance clarity and meaning	a Revise own writing to enhance clarity and meaning	a Revise own writing to enhance clarity and meaning	a Revise own writing to correctiveness and comprehensiveness
b Use descriptive terms to emphasize facts and quantities	b Use descriptive terms to emphasize facts and quantities	b Use descriptive and connecting terms to enhance meaning, clarity and precision	b Use descriptive and connecting terms to enhance meaning, clarity and precision	b Use descriptive and connecting terms to enhance meaning, clarity and precision

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Edit for complete and correct sentences, punctuation and usage	a Edit for complete and correct sentences, punctuation and usage	a Edit for complete and correct sentences, punctuation and usage	a Edit for complete and correct sentences, punctuation and usage	a Edit to produce a correct legible, effective piece of writing
b Spell correctly	b Spell correctly	b Spell correctly	b Spell correctly	b Spell correctly
c Produce legible final copy	c Produce legible final copy (manual or electronic processes)	c Produce legible final copy (manual or electronic processes)	c Produce legible final copy (manual or electronic processes)	c Produce legible final copy (manual or electronic processes)

9.0 Appropriate Study Skills: The student selects and uses appropriate study skills in order to accomplish mathematical learning tasks.

9.1 IDENTIFY MAIN IDEAS, SUPPORTING DETAILS, AND FACTS AND OPINIONS PRESENTED IN WRITTEN, ORAL AND VISUAL FORMATS (ELS 2.1)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
[a] Locate facts in oral and visual formats	[a] Locate facts in oral and visual formats	[a] Locate facts in grade-level materials	[a] Locate facts in grade-level materials	[a] Locate facts in grade-level selections
				[b] Recall facts and supporting evidence
			[c] Identify main idea in a problem situation	[c] Identify main idea in a problem situation

9.2 USE INSTRUCTIONAL MATERIALS AS BASIS FOR GAINING KNOWLEDGE AND IMPROVING COMPREHENSION (ELS 2.2)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
	[a] Use table of contents to locate general information	[a] Use table of contents to locate general information	[a] Use table of contents to locate general and specific information	[a] Use table of contents and index to locate general and specific information
				[b] Use supportive illustrations, detail and summations to obtain information (e.g., captions, footnotes, glossary entries, graphs, tables, charts, maps)
			[c] Use guide words in a dictionary or glossary to locate words	[c] Use guide words and diacritical markings to locate and pronounce words

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GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Locate facts in grade-level selections	a Locate facts in grade-level selections	a Locate facts in grade-level selections	a Locate facts in grade-level selections	a Locate facts in grade-level selections
b Recall facts and supporting evidence	b Identify necessary and extraneous facts and related supporting details	b Identify necessary and extraneous facts and related supporting details	b Identify necessary and extraneous facts and related supporting details	b Identify necessary and extraneous facts and related supporting details
c Identify main idea in a problem situation	c Identify main idea in a problem situation	c Identify main idea in a problem situation	c Identify main idea in a problem situation	c Identify main idea in a problem situation

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Use table of contents and index to locate general and specific information	a Use table of contents, index, and summaries to locate information needed	a Use table of contents, index, summaries, charts and graphs to locate information needed	a Use table of contents, index, summaries, charts, graphs and illustrations to locate information needed	a Use table of contents, index, summaries, charts, graphs and illustrations to locate information needed
b Use supportive illustrations, detail and summations to obtain information	b Use summaries and headings	b Use summaries and headings	b Use organization of materials (summaries, headings and review questions)	b Use organization of materials (summaries, headings and review questions)
c Use diacritical markings or respellings to pronounce words	c Use diacritical markings or respellings to pronounce words	c Use diacritical markings or respellings to pronounce words	c Use diacritical markings or respellings to pronounce words	c Use diacritical markings or respellings to pronounce words

9.3 CLARIFY PURPOSES OF ASSIGNMENT (ELS 7.1)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Repeat oral instructions	a Repeat oral instructions	a Repeat oral instructions in proper sequence and ask questions to clarify	a Determine general purpose of assignment and ask clarification on questions if necessary b Determine ideas and concepts addressed in the assignment	a Determine general purpose of assignment and ask clarification on questions if necessary b Determine ideas and concepts addressed in the assignment

9.4 USE RESOURCES BEYOND THE CLASSROOM (ELS 7.2)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
				a Locate, check-out and return books and other circulating media materials b Locate and use noncirculating reference materials c Use library classification system and services to locate specialized resources required to complete assignments

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Determine general purpose of assignment and ask clarification on questions if necessary	a Determine general purpose of assignment and ask clarification on questions if necessary	a Determine general purpose of assignment and ask clarification on questions if necessary	a Determine general purpose of assignment and ask clarification on questions if necessary	a Determine general purpose of assignment and ask clarification on questions if necessary
b Determine ideas, concepts, and generalities addressed in the assignment	b Determine ideas, concepts, and generalities addressed in the assignment	b Determine ideas, concepts, and generalities addressed in the assignment	b Determine ideas, concepts, generalities or principles included in assignment	b Determine ideas, concepts, generalities or principles included in assignment

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Locate, check-out and return books and other circulating media materials	a Locate, check-out and return books and other circulating media materials	a Locate, check-out and return books and other circulating media materials	a Locate, check-out and return books and other circulating media materials	a Locate, check-out and return books and other circulating media materials
b Locate and use non-circulating reference materials	b Locate and use non-circulating reference materials	b Locate and use non-circulating reference materials	b Locate and use non-circulating reference materials	b Locate and use non-circulating reference materials
c Use library classification system and services to locate specialized resources required to complete assignments	c Use library classification system and services to locate specialized resources required to complete assignments	c Use library classification system and services to locate specialized resources required to complete assignments	c Use library classification system and services to locate specialized resources required to complete assignments	c Use library classification system and services to locate specialized resources required to complete assignments
		d Use computer (e.g., data bases, spread sheets)	d Use computer (e.g., data bases, spread sheets)	d Use computer (e.g., data bases, spread sheets)

9.5 SELECT AND USE APPROPRIATE STUDY TECHNIQUES (ELS 7.3)

KINDERGARTEN	GRADE 1	GRADE 2	GRADE 3	GRADE 4
a Select activities and use time effectively	a Select activities and use time effectively	a Follow a study plan including: time management, appropriate study environment, processing of information	a Follow a study plan including: time management, appropriate study environment, processing of information	a Follow a study plan including: time management, appropriate study environment, processing of information
b Begin and complete task	b Begin and complete assignment and ask questions to clarify	b Begin and complete assignment and ask questions to clarify	b Accomplish learning task using appropriate study techniques (read and reread text, ask clarifying questions, seek help when needed, use memory techniques)	b Accomplish learning task using appropriate study techniques (e.g., read and reread text, ask clarifying questions, seek help when needed, use memory techniques and study with classmates)
			c Vary reading rate according to purpose for reading the selection	c Vary reading rate according to purpose for reading the selection
d Keep materials organized and accessible	d Keep materials organized and accessible	d Keep materials organized and accessible	d Keep study materials organized and accessible	d Keep study materials organized and accessible
e Complete a task within a given amount of time	e Turn in assignments on time	e Turn in assignments on time	e Turn in assignments on time	e Turn in assignments on time
	f Use appropriate test-taking techniques	f Use appropriate test-taking techniques	f Use appropriate test-taking techniques	f Use appropriate test-taking techniques

GRADE 5	GRADE 6	GRADE 7	GRADE 8	GRADE 11
a Follow a study plan including: time management, appropriate study environment, processing of information	a Follow a study plan including: goal setting, time management, appropriate study environment, processing of information	a Follow a study plan including: goal setting, time management, appropriate study environment, processing of information	a Follow a study plan including: time management, appropriate study environment, processing of information	a Follow a study plan including: time management, appropriate study environment, processing of information
b Accomplish learning task using appropriate study techniques (preview and review chapters, read and reread text, ask clarifying questions, seek help when needed, use memory techniques, summarize, study with classmates, use self-questioning)	b Accomplish learning task using appropriate study techniques (preview and review chapters, read and reread text, ask clarifying questions, seek help when needed, use memory techniques, summarize, study with classmates, use self-questioning)	b Accomplish learning task using appropriate study techniques (preview and review chapters, read and reread text, ask clarifying questions, seek help when needed, use memory techniques, summarize, study with classmates, use self-questioning)	b Accomplish learning task using appropriate study techniques (preview and review chapters, read and reread text, ask clarifying questions, seek help when needed, use memory techniques, summarize, study with classmates, use self-questioning)	b Accomplish learning task using appropriate study techniques (preview and review chapters, read and reread text, ask clarifying questions, seek help when needed, use memory techniques, summarize, study with classmates, use self-questioning)
c Vary reading rate according to purpose for reading the selection	c Vary reading rate according to purpose for reading the selection	c Vary reading rate according to purpose for reading the selection	c Vary reading rate according to purpose for reading the selection	c Vary reading rate according to purpose for reading the selection
d Keep study materials organized and accessible	d Keep study materials organized and accessible	d Keep study materials organized and accessible	d Keep study materials, log and related notes organized and accessible	d Keep study materials, log, related notes and filing system organized and accessible
e Turn in assignments on time	e Turn in assignments on time	e Turn in assignments on time	e Turn in assignments on time	e Turn in assignments on time
f Use appropriate test-taking techniques	f Use appropriate test-taking techniques	f Use appropriate test-taking techniques	f Use appropriate test-taking techniques	f Use appropriate test-taking techniques

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BIBLIOGRAPHY

Ashlock, Robert B. Error Patterns in Computation, A Semi-Programmed Approach, Fourth Edition. Columbus, OH: Charles E. Merrill Publishing Company, 1986.

The reader is engaged in a careful analysis of systematic errors in computation; the ability to identify the probable conceptual or procedural bases for errors is explicitly promoted, as is the ability to decide what directions are appropriate for follow-up instruction.

Baratta-Lorton, Mary. Mathematics Their Way. Menlo Park, CA: Addison-Wesley, 1976.

The goal of the activities in this book is to develop understanding and insight of the patterns of mathematics through use of concrete materials. There are both total-group and small-group experiences for primary school children. Each activity includes a list of the specific skills taught and the materials needed, as well as a description of the activity itself.

Baratta-Lorton, Robert. Mathematics, A Way of Thinking. Meno Park, CA: Addison-Wesley, 1977.

The purpose of this book is to provide a detailed guide for teachers wishing to implement an activity-centered approach to the teacher of mathematics. The book may be used as a complete mathematics curriculum for upper elementary or as a resource to teachers wishing to supplement textbook lessons with manipulative activities.

Brown, Stephen and Marion Walter. The Art of Problem Posing. Palo Alto, CA: Dale Seymour Publications.

Attention is given to help the reader understand what problem solving consists of and why it is important. Strategies for engaging in and improving problem solving are presented as a means of improving problem solving skills. Many examples from secondary school mathematics are used to illustrate the points the authors have in mind.

Burns, Marilyn. Meeting the Math Challenge. Palo Alto, CA: Learning Institute, 1982.

A handbook for leaders of mathematics workshops for teachers interested in implementing the NCTM's Agenda for Action. A strong emphasis is placed upon individual and group problem solving, cooperative learning, and the appropriate classroom environment. Many activities from elementary mathematics are used as illustrations.

Burns, Marilyn. The Math Solution, Teaching for Mastery Through Problem Solving. Sausalito, CA: Marilyn Burns Education Association, 1984.

These K-8 supplementary materials are designed to help make problem solving the focus of mathematics curriculum. Mathematics is taught as a way of thinking--a tool for solving problems.

Copeland, Richard W. How Children Learn Mathematics: Teaching Implications of Piaget's Research. New York: The Macmillan Company, 1970.

This book is intended to help teachers discern a child's stage of development as a basis for determining type of mathematics for which he or she is ready. It summarizes the ideas of Piaget in a form most teachers will find readable.

Driscoll, Mark. Research Within Reach: Elementary School Mathematics. National Institute of Education, U.S. Department of Education, Washington, DC 20208. (Also available through the National Council of Teachers of Mathematics.)

A research-guided response to concerns of educators, including bridging the gap between the concrete and abstract through the use of manipulatives, models, drawings, patterns. Also presents research findings on questions relating to drill, teaching basic facts and algorithms, calculators, diagnosis, evaluation, grouping, motivation.

Driscoll, Mark. Research Within Reach: Secondary School Mathematics. National Institute of Education, U.S. Department of Education, Washington, DC 20208. (Also available through the National Council of Teachers of Mathematics.)

A research-guided response to concerns of educators, including effective mathematics teaching, communicating mathematics, breaking the vicious cycles, remediation in secondary school mathematics, problem solving, understanding fractions, estimation, learning and teaching algebra, and learning and teaching geometry.

Driscoll, Mark and Jere Comfrey. Teaching Mathematics: Strategies That Work; Teacher Writing to Teacher Series. Chelmsford, MA: Northwest Regional Exchange, Inc.

Articles written by a group of talented and enthusiastic professionals to their colleagues about pedagogy in teaching mathematics.

Fennema, Elizabeth. Mathematics Education Research: Implications for the 80s. Association for Supervision and Curriculum Development, 1981.

This publication is a collection of articles by authors who have synthesized current research in mathematics education. Their observations include findings on such areas as problem solving, calculators, computers, factors related to gender, and teacher/student interaction characteristics.

Hirsch, Christian R. Activities for Implementing Curricular Themes From the Agenda for Action. Washington, DC: National Council of Teachers of Mathematics, 1986.

Selections from the Mathematics Teacher which can be used to implement problem solving, expanding basic skills, and using calculators, computers, and manipulatives.

Hirsch, Christian R. The Secondary School Mathematics Curriculum (Yearbook; 1985). Washington, DC: National Council of Teachers of Mathematics, 1985.

This yearbook is a collection of forward-looking articles addressing the issue of what the secondary mathematics curriculum should be in order to prepare our students for the life they face in an ever-changing technological society.

Labinowicz, Ed. Learning From Children: New Beginnings for Teaching Numerical Thinking. Menlo Park, CA: Addison-Wesley, 1985.

This book attempts to build a bridge between theory, research, and classroom practice with the aim of supporting informed change in schools. The research reported is characterized by clinical interviews of individual children and observation of the teaching-learning process through experiments with small groups of children.

Labinowicz, Ed. Learning From Children. New Beginnings for Teaching Numerical Thinking. Menlo Park, CA: Addison-Wesley Publishing Company, 1985.

Specific suggestions from the Piagetian point of view for teaching the concepts and operations with numbers.

Labinowicz, Ed. The Piaget Primer--Thinking, Learning, Teaching. Menlo Park, CA: Addison-Wesley Publishing Company, 1980.

This book presents the essence of Piaget's ideas with a minimum of technical vocabulary. It also bridges the gap between his theory, his research methods, and classroom practice in presenting mathematics and science concepts.

Lane County Mathematics Project. Problem Solving in Mathematics. Eugene, OR: Lane Education Service District, 1983. (Available from Dale Seymour Publications, PO Box 10888, Palo Alto, CA 94303-0879.)

The activity-oriented materials provide supplementary units on problem solving for grades four through eight and algebra. A full range of problem-solving skills is included.

Lappan, Glenda. Middle Grades Mathematics Project. Michigan State University: Addison-Wesley Publishing Company, 1986.

Excellent two-three week units utilizing manipulation to teach units, such as Probability, Factors and Multiples, Spacial Visualization, Simulation, Mouse and Elephant.

Laycock, Mary and Gene Watson. The Fabric of Mathematics: A Resource Book for Teachers (Revised Edition). Hayward, CA: Activity Resources Company, Inc., 1975.

This resource book contains many suggested classroom activities for teachers interested in using concrete materials to introduce and reinforce elementary mathematics concepts.

Laycock, Mary and Margaret A. Smart. Hands On Math for Secondary Teachers. Hayward, CA: Activity Resources Company, Inc., 1984.

This book was written to help fill the need of the secondary teacher who wants alternative strategies in teaching basic mathematics to junior and senior high students. This book will help you integrate manipulatives into your math program and help your students gain understanding of mathematics at a concrete level.

Lester, Frank K. and Joe Ganofalo. Mathematical Problem Solving: Issues in Research. Philadelphia PA: The Franklin Institute Press, 1982.

Papers of leading researchers of mathematical problem solving are presented, including a chapter on "Building Bridges Between Psychological and Mathematics Education Research on Problem Solving."

Marcy, Janice and Steve Marcy. Math, Imagination, and Pizzazz Series. Palo Alto, CA: Creative Publications, 1978.

Presents puzzles at many skill levels.

Mason, John with Leone Burton and Kaye Stacey. Thinking Mathematically. London, England: Addison-Wesley Publishing Company, 1982.

The authors believe that mathematical thinking can be improved. They present their theory and make their ideas explicit with the use of problems in which the reader is expected to seek appropriate solutions.

Math and the Mind's Eye. Available (beginning October 1987) from the Math Learning Center, PO Box 3226, Salem, OR 97302.

A collection of 38 activities for grades 5 to 9 utilizing visual methods for learning mathematics. The activities are separated into five units: I Seeing Mathematical Relationships, II Visualizing Number Concepts, III Modeling Whole Numbers, IV Modeling Fractions and Decimals, V Looking at Geometry.

Mathematics Resource Project. Geometry and Visualization. Eugene, OR: Department of Mathematics, University of Oregon, 1976. (Available from Creative Publications, PO Box 10328, Palo Alto, CA 94303.)

This lengthy collection of classroom materials for middle school students contains units on Lines, Planes and Angles; Polygons and Polyhedra; Curves and Curved Surfaces; Similar Figures; and Area and Volume. Teaching emphases of the materials include visual perception, graphic representation, calculators, applications, estimation and approximation, and laboratory approaches.

Mathematics Resource Project. Mathematics in Science and Society. Eugene, OR: Department of Mathematics, University of Oregon, 1976. (Available from Creative Publications, PO Box 10328, Palo Alto, CA 94303.)

This collection of classroom materials for middle school students is organized into six units: Mathematics and Astronomy, Mathematics and Biology, Mathematics and Environment, Mathematics and Music, Mathematics and Physics, and Mathematics and Sports. Problem solving, teaching for transfer, and laboratory approaches are stressed.

Mathematics Resource Project. Number Sense and Arithmetic Skills. Eugene, OR: Department of Mathematics, University of Oregon, 1975. (Available from Creative Publications, PO Box 10328, Palo Alto, CA 94303.)

This collection of classroom materials for middle school students contains resource ideas on whole number operations, numeration systems, fractions, and decimals. Teaching emphases of the materials include calculators, applications, problem solving, mental arithmetic, algorithms, estimation and approximation, and laboratory approaches.

Mathematics Resource Project. Ratio, Proportion and Scaling. Eugene, OR: Department of Mathematics, University of Oregon, 1975. (Available from Creative Publications, PO Box 10328, Palo Alto, CA 94303.)

This collection of classroom materials for middle school students contains resource ideas on the topics, ratio, proportion, scaling, and percent. Teaching emphases of the materials include calculators, applications, problem solving, mental arithmetic, estimation and approximation, and laboratory approaches.

McFadden, Scott, Keith Anderson, and Oscar Schaaf. Math Lab--Junior High. Eugene, OR: Action Math Associates, Inc., 1975.

This book consists of pages which can be reproduced to make activity cards presenting nine topic strands, including perimeter and area, volume, calculators, and applications. The tasks presented are problem solving in nature and many require hands-on investigations.

McFadden, Scott, Judy Johnson, Mary Ann Todd, and Oscar Schaaf. Math Labmetric. Eugene, OR: Action Math Associates, Inc., 1976.

This series of three books (separate editions for grades 1-2, 3-4, 5-6) contains numerous activity cards for use in elementary math classrooms. Fifteen math topics are covered at each grade level, and the tasks presented are activity based.

McFadden, Scott. Algebra Warm-Ups. Short Exercises for Review and Exploration. Palo Alto, CA: Dale Seymour Publications, 1987.

Activities for the first few minutes of class with emphasis on skill maintenance, concept development, and problem solving. Content includes topics from first year algebra.

McFadden, Scott. Math Warm-Ups for Junior High (7-9). Short Exercises for Review Exploration. Palo Alto, CA: Dale Seymour Publications, 1983.

Activities for the first few minutes of class with emphasis on skill maintenance, concept development, mental computation, and problem solving. Content areas include whole numbers, fractions, decimals, money, percent, measurement, and geometry.

National Council of Teachers of Mathematics, Inc. The Arithmetic Teacher. Reston, VA: National Council of Teachers of Mathematics, Inc.

This magazine is one of two official journals of the National Council of Teachers of Mathematics. It is published nine times yearly and is distributed to NCTM members. It contains a wealth of ideas that can be put to use in the elementary and middle school mathematics classroom.

National Council of Teachers of Mathematics, Inc. The Mathematics Teacher. Reston, VA: National Council of Teachers of Mathematics, Inc.

This magazine is one of two official journals of the National Council of Teachers of Mathematics. It is published nine times yearly and distributed to NCTM members. It contains information on recent happenings in secondary mathematics education and includes a special activities section each issue.

Oregon Council of Teachers of Mathematics. The Oregon Mathematics Teacher. Portland, OR: Oregon Council of Teachers of Mathematics.

This magazine is the official journal of the Oregon Council of Teachers of Mathematics. In contains many classroom-tested ideas for use in both elementary and secondary schools. The journal is available through membership in OCTM; for information, contact, Specialist, Math Education, Oregon Department of Education.

Oregon Department of Education. MATH in Oregon Schools. Salem, OR: Department of Education, 1977.

Oregon Department of Education. The Oregon Mathematics Concept Papers. Salem, OR: Department of Education, 1986.

These position papers were written by teachers for teachers. The information and recommendations presented are based on a study of research, reports, and recommendations appearing in the national literature. The eight papers address forward-looking issues at each school level.

Polya, G. How to Solve It. Princeton, NJ: Princeton University Press, 1973.

This book contains a comprehensive discussion of the nature of problem solving and suggests many specific problem-solving strategies.

Reys, Robert E., Marilyn N. Suydam, and Mary M.Lindquist. Helping Children Learn Mathematics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1984.

This book is for teachers of mathematics in elementary school. It is written to help teachers help children learn important mathematical concepts, skills, and problem-solving techniques. The first part of the book provides a base for understanding the changing mathematics curriculum and how children learn. The second part discusses teaching strategies, techniques, and learning activities.

Reys, Robert E. Helping Children Learn Mathematics. Englewood Cliffs, NJ: Prentice Hall, Inc., 1984.

Presents a basic overview of background using technology.

Richardson, Kathy. Developing Number Concepts Using Unifix Cubes. Menlo Park, CA: Addison-Wesley, 1984.

This book is for teachers who want to make mathematics understandable for their students. The concepts dealt with range from beginning counting to the introduction of multiplication and division. The book is based on the premise that children are taught mathematics so it can serve as a useful, problem-solving tool in their lives.

Romberg, Thomas A. "A Common Curriculum for Mathematics." Individual Differences and the Common Curriculum Yearbook, University of Chicago: National Society for the Study of Education, 1983.

A case is made for learning by doing mathematics. Four related activities common to all of mathematics are identified: abstracting, inventing, proving, and applying. These activities are the natural consequences of mathematical investigations and can be made an integral part of school mathematics instruction.

Schminke, C. W. and Enosh Dumas. Math Activities for Child Involvement, Third Edition. Boston MA: Allyn and Bacon, Inc., 1981.

Children learn best when they are stimulated, surprised, amused, and are successful. The activities emphasize concept development, basic skill development, and extended understanding. Context includes prenumber topics, numeration and place value, number operations with whole numbers and fractions, geometry, and measurement.

Schoen, Harold L. Estimation and Mental Computation (Yearbook; 1986). National Council of Teachers of Mathematics, 1986.

This yearbook is a response to the growing recognition among mathematics educators of the importance of teaching and learning estimation. Its articles make a strong case for the importance of estimation in various domains such as numerosity, computation, and measurement.

Sharron, Sidney. Applications in School Mathematics (Yearbook; 1979). National Council of Teachers of Mathematics, 1979.

Much of this yearbook deals with forming and using mathematical models to make mathematics more meaningful to students. It also addresses the problem that many teachers have incorporating real-life applications in their instruction since most of their training has dealt with the abstract rather than the practical or serviceable areas of mathematics.

Sobel, Max A. and Evan M. Maletsky. Teaching Mathematics: A Sourcebook of Aids, Activities, and Strategies. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1975.

This book presents a collection of experiments and activities for secondary mathematics classrooms. Also included is a bibliography of resources and references.